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A new analytical solution for prediction of forward tension in the drawing process

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ABSTRACT

Tension in the drawing process is determined via a new solution to the drawing problem for dies of axisymmetric or symmetric sections. A free body equilibrium method is used to obtain the equations that dictate the drawing phenomenon. As opposed to the classical slab method, solution of these equations accounts for internal material distortion. An analytical iterative equation is used to solve the problem of tensions. The new solution can be applied to dies of either axisymmetric or symmetric sections by simply varying one of the constants used in the model. The results afforded by this solution agree with data from experiments and simulations performed using other methods; it correctly reproduces experimental data and behavioural trends. There is also a high degree of similarity between the results obtained with the analytical iterative equation and those obtained with the finite elements method (FEM).

1. Introduction

The drawing process is a forming process for plastic deformation of industrial metals, and is especially used in the electric sector. It consists of reducing (Ghaei et al., 2005; Avitzur, 1987) or changing the shape of the cross-section, or a combination of both (Kim et al., 1999), whereby the material is run through a die that defines its final shape. The main variables involved in this process comprise the die angle α, the cross-section A, the friction coefficient μ, the area reduction r, and the yield tension σ_x (see Fig. 1). It differs from other plastic forming processes primarily in that the traction tension applied to the working material is limited, and that the maximum tension allowed on the material section during drawing is equal to the yield tension. The drawing process is characterised by two factors: a limit on the reduction that occurs during drawing; consumption of a fraction of the process potential by the frictional forces between the die wall and the material.

Various analytical, empirical, numerical and experimental methods have been developed to predict the drawing tension and to determine the best combination of process parameters. The most common analytical methods for evaluation and simulation of drawing are homogenous deformation, the slab method, and the upper bound technique (Hosford and Caddell, 1993).

We have developed a drawing tension model based on a free body equilibrium approach, which entails force balance on a slab metal of differential thickness. This technique yields ordinary differential equations, in which the dependent variables are a function of a unique spatial coordinate, among other, independent variables. What distinguishes this method from the classical slab method is the introduction of the effects of internal material distortion produced by surface friction into the differential equation which governs the problem, and into its solution. This distortion has an influence on the orientation of the principal directions of the material. It is introduced into

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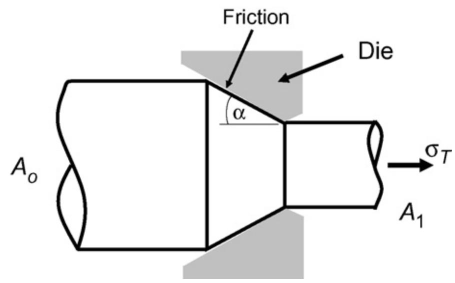

Fig. 1 – Axisymmetric die and wire (not to scale).

the model via the shear tension τ, which is considered sizeable in the von Mises equation, but is considered negligible in the classical slab method.

Several research articles on the design of extrusion dies have analysed this problem, focusing on optimisation of the variable strain rate to obtain a die geometry that minimises this variable (Joun and Hwang, 1993; Lu and Lo, 1999). Lo and Lu (2002) obtained a partial differential equation which is a function of the velocity field and the shear stress, and which incorporates internal distortion of the material. To solve their equation, they assumed that the velocity distributions are known, thereby reducing it into an ordinary differential equation that can be solved numerically. In contrast to the model developed by Lo, which is a function of the velocity field, our model is a function of the tension field.

The following sections describe the mathematical model formed by the differential equations that govern the drawing process as well as the different yield criteria employed. The third section outlines the algorithm for solution and simulation. The first part of the fourth section describes validation of the model by comparing the drawing force results with those of other models, such as the classical slab and the finite element methods. It also provides a comparison of the respective solutions obtained from experimental data.

2. Mathematical model

The model to predict the forward tension in the drawing process is obtained by balancing on a differential element, as shown in Fig. 2. The differential equation associated with the forces problem is linearised, and then solved analytically to generate a function for the tension.

In obtaining the drawing process model, the following assumptions are made:

(i) The die is considered a rigid body, and the drawing material is considered a rigid-plastic material.
(ii) The plastic deformation is plane strain.
(iii) The averaged stresses are uniformly distributed within the elements.
(iv) There is friction at the die–material interface, and the dynamic friction coefficient is constant.
(v) The material flows into and out of the system horizontally.
(vi) The die angle is small.

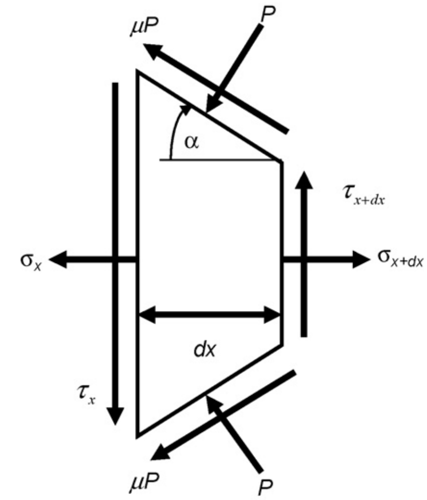

Fig. 2 – Material element.

The model obtained represents forward tension as a function of the area reduction r, the friction coefficient μ, the die angle α, the yield shear stress k, and the die length x.

2.1. *Equilibrium equations for symmetric and axisymmetric dies*

The equilibrium equations for the differential element (see Fig. 2) are obtained by balancing the forces in the horizontal direction:

$$\sum F_x = 0 \Rightarrow \frac{\partial \sigma_x}{\partial x} + \frac{n}{R(x)}\left[\sigma_x \frac{\partial R(x)}{\partial x} - P[\tan(\alpha) + \mu]\right] = 0 \qquad (1)$$

In this equation, the shear stress resulting from the friction between the die and the material is considered to be equal to the product of the dynamic friction coefficient μ and the pressure P. Thus it is also assumed that this shear stress is proportional to the normal pressure to which the material is subjected. The constant n allows generalization of the equations for balancing forces: if $n = 1$, then the problem corresponds to symmetric plane deformation; but if $n = 2$, then the problem corresponds to an axisymmetric plane deformation. The variables and constants that appear in (1) are: the tension in the direction x, σ_x; the radius or thickness (depending on the value chosen for n) $R(x)$; the pressure P that the die places on the material; the die angle α, which is treated as constant. The die is assumed to be conical; hence the radius or thickness of the material in the deformation zone is a linear function:

$$R(x) = R_0 - x\tan(\alpha) \qquad (2)$$

whereby R_0 is the initial radius or thickness of the material.

A straightforward method for adimensionalisation is to set the length of the die to one, for which the following equation is used:

$$x = \frac{R_0 \cdot r}{n\tan(\alpha)} x* \qquad (3)$$

whereby x^* is the adimensionalised position x.

For conical dies, the slope defined by the contact zone between the die and the material is

$$\frac{\partial R}{\partial x} = -\tan(\alpha) \quad (4)$$

The differential Eq. (1) then becomes

$$\frac{\partial \sigma_x}{\partial x*} + \frac{r}{(1-(r/n)x*)\tan(\alpha)}[-\sigma_x \tan(\alpha) - P[\tan(\alpha)+\mu]] = 0 \quad (5)$$

whereby r is the area reduction (6), which is a function of the sections of the material at the entrance and the exit of the die, denoted by A_0 and A_1, respectively:

$$r = \frac{A_0 - A_1}{A_0} \quad (6)$$

Solving Eq. (5) requires a function that relates the pressure P to the drawing tension σ_x, and which is assumed to be linear:

$$P = A - B\sigma_x \quad (7)$$

whereby A and B are constants to be calculated.

Introducing (7) into (5) yields an ordinary differential equation, which is solved by using the border condition (8) in Eq. (9):

$$\sigma_x|_{x*=0} = 0 \quad (8)$$

This equation relates the drawing tension to the independent variables that govern wiredrawing: the die angle α; the dynamic friction coefficient μ; the area reduction r; the adimensional position within the die x^*.

$$\sigma_x = \frac{A(1+(\mu/\tan(\alpha))}{B(1+(\mu/\tan(\alpha))-1}\left[1-\left(1-\frac{r}{n}x*\right)^{n[B(1+(\mu/\tan(\alpha))-1]}\right] \quad (9)$$

2.2. *Yield criterion*

Normal and shear tensions are related in a differential element by using the von Mises yield criterion. Assuming that the drawing problem is a plane strain problem, and that the increase in plastic deformation depends on the deviatoric stress (Oller, 2001), then the von Mises yield criterion (10) can be used as a criterion for plastic fluidity or discontinuity (Kachanov, 2004):

$$\left|\sigma_x - \sigma_y\right| = 2k\sqrt{1-4\left(\frac{\tau}{2k}\right)^2} \quad (10)$$

whereby k is the yield limit for a pure shear.

The following sections describe three approaches to determine the yield criterion.

2.2.1. *The first approach to yield criterion: the classical slab method*

In this method, the pressure P produced by the die is related to the normal tension σ_x through Eq. (11). This implies the assumption that the tension σ_y is approximately equal to $-P$

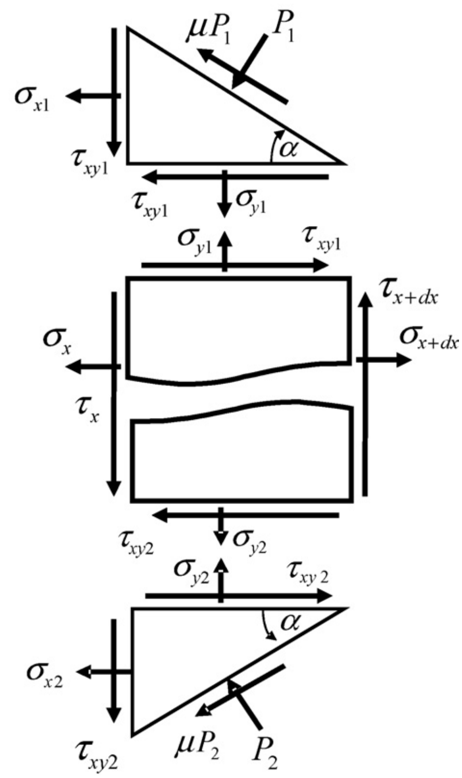

Fig. 3 – Stress state for infinitesimal triangular elements.

and that the shear tension is negligible:

$$P = 2k - \sigma_x \quad (11)$$

Therefore A is made equal to $2k$ (the yield limit for tension), and B is made equal to 1. Substituting these constants into the general Eq. (9) affords the classical equation for drawing tension:

$$\sigma_x = \frac{2k(\mu+\tan(\alpha))}{\mu}\left[1-\left(1-\frac{r}{n}x*\right)^{n\mu/\tan(\alpha)}\right] \quad (12)$$

2.2.2. *The second approach to yield criterion*

In this method, the pressure P produced by the die is related to the normal tension σ_x through a force balance equation calculated over the differential triangular elements shown in Fig. 3. The force balance on the highest of these elements yields the following equations:

$$\sum F_x = 0 \Rightarrow \tau_{xy1} + (\sigma_{x1} + P_1)\tan(\alpha) + \mu P_1 = 0 \quad (13)$$

$$\sum F_y = 0 \Rightarrow -\sigma_y + (\mu P_1 - \tau_{xy1})\tan(\alpha) - P_1 = 0 \quad (14)$$

Eliminating τ_{xy1} with (13) and (14), gives

$$-(\sigma_{y1} + P_1) + (\sigma_{x1} + P_1)\tan^2(\alpha) + 2\mu P_1 \tan(\alpha) = 0 \quad (15)$$

Likewise, the force balance on the lowest of these elements yields the following equation:

$$-(\sigma_{y2} + P_2) + (\sigma_{x2} + P_2)\tan^2(\alpha) + 2\mu P_2 \tan(\alpha) = 0 \quad (16)$$

Taking the sum of (15) and (16), and assuming that σ_y is equal to one half the sum of σ_{y1} and σ_{y2}, and likewise for σ_x and P, gives

$$\sigma_x - \sigma_y = (\sigma_x + P)(1 - \tan^2(\alpha)) - 2\mu P \tan(\alpha) \quad (17)$$

By substituting (17) in (10), and assuming that the shear tension τ is negligible, the second yield criterion is obtained:

$$P = \frac{2k}{1 - \tan^2(\alpha) - 2\mu\tan(\alpha)} - \frac{1 - \tan^2(\alpha)}{1 - \tan^2(\alpha) - 2\mu\tan(\alpha)}\sigma_x \quad (18)$$

whereby the first term of the Eq. (18) corresponds to A, and the multiplier factor for σ_x corresponds to B.

Substituting these constants into the general solution (9) yields a new approximation of the drawing tension.

2.2.3. *The third approach to yield criterion*

This method is characterised above all by the fact that the shear stress has a non-zero value in Eq. (10). This is achieved by calculating the shear tension τ_{xy1} from (13) and (14) to obtain the following equation:

$$\tau_{xy1}(1+\tan(\alpha)) = -\sigma_{x1}\tan(\alpha) - \sigma_{y1} + P_1(\tan(\alpha) - \mu - 1 + \mu\tan(\alpha)) \quad (19)$$

τ_{xy2} is calculated the same way. Assuming that τ_{xy} is equal to one half the sum of τ_{xy1} and τ_{xy2}, gives the following equation for shear stress:

$$\tau_{xy} = \frac{P\tan(\alpha)}{1 + \tan(\alpha)} \quad (20)$$

The von Mises equation with non-zero τ (10) is strongly non-linear. However, provided that τ is less than $2k$, then a good linear approximation of (10) can be obtained. Assuming that this premise is true, then calculation of the Taylor series yields the following linear approximation of (10):

$$|\sigma_x - \sigma_y| = 2k\left[1 - 2\left(\frac{\tau}{2k}\right)^2\right] \quad (21)$$

Substituting (17) and (20) into (21), and linearising the equation via Picard's method, provides the third yield criterion (22). In agreement with approximation (22), the pressure P is determined by estimating the pressure P^{i-1}. Therefore P^i is the solution to the problem:

$$P^i = \frac{2k}{1 - \tan^2(\alpha) - 2\mu\tan(\alpha) + 2(P^{i-1}/2k)(\tan^2(\alpha)/(1+\tan(\alpha))^2)} - \frac{1-\tan^2(\alpha)}{1-\tan^2(\alpha)-2\mu\tan(\alpha)+2(P^{i-1}/2k)(\tan^2(\alpha)/(1+\tan(\alpha))^2)}\sigma_x^i \quad (22)$$

whereby P^{i-1} is the pressure at the iteration $(i-1)$. Again, the first term of Eq. (22) corresponds to A, and the multiplier factor for σ_x^i corresponds to B. Substituting these constants into the general solution (9) provides the new approach to determining the drawing tension.

3. Solution algorithm

We developed a simple programme to solve the drawing equation which employs a Picard-type (Kelley, 1995) iterative algorithm. The zero iteration of this algorithm corresponds to a problem in which the shear stress τ is zero in the von Mises criterion (10). In contrast, the subsequent iterations incorporate the shear tension in the criterion (10), evaluating it as a function of the pressure P^{i-1}, which is the pressure calculated in the preceding iteration. The algorithm is shown below:

Let i=0
Read initial pressure condition $P_0=0$
Evaluating the constants A and B of the equation (18)
Evaluating σ_x^0 of the equation (9)
Evaluating P^0 of the equation (18)
While not convergence
i←i+1
Evaluating constants A and B of the equation (22)
Evaluating σ_x^i of the equation (9)
Evaluating P^i of the equation (22)
Evaluating the convergence
End while

The convergence is verified by applying the Euclidean discrete norm, calculating the difference in absolute value between the unknown drawing tension σ_x^i and the known tension σ_x^{i-1}. The value of the Euclidean discrete norm must be less than a given ε. Empirically, we have found that after the fifth iteration, the value of ε is less than 1%. The graphs shown later in this work were all based on five iterations.

4. Results

In this section, results obtained from different simulations are reported, and validated against different analytical, numerical and experimental solutions. Specifically, the new solution (the third approximation of the yield criterion) is compared with the classical slab method (the first approximation), the second approximation, and the finite element method (FEM). Lastly, the model results are compared with experimental data reported by Wistreich (1955).

Fig. 4 shows the dimensionless drawing tension in function of the die angle, for symmetric geometry ($n=1$), a reduction r of 0.2, and a friction coefficient μ equal to 0.1 and to 0.2. The figure reveals four distinct curves which correspond to the four solutions obtained via the different methods: the continuous thick line corresponds to the third yield criterion; the dashed thick line, to the numerical solution obtained with the FEM

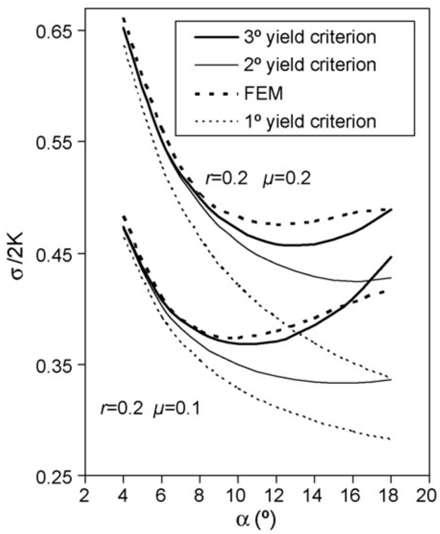

Fig. 4 – Dimensionless drawing tension for symmetric dies.

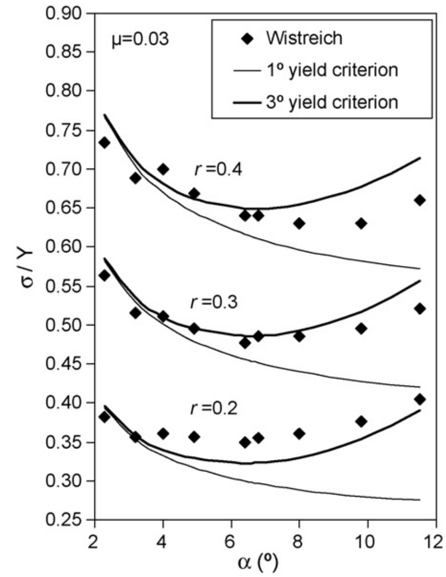

Fig. 6 – Comparison of experimental and simulation values for dimensionless drawing tension, for axisymmetric dies.

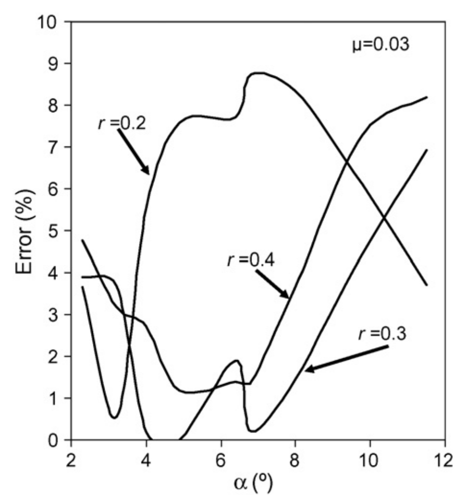

Fig. 7 – Percent error in the dimensionless drawing tension for values obtained experimentally and from the third yield criterion, for axisymmetric dies.

(Rubio et al., 2005); the continuous thin line, to the second yield criterion; the dashed thin line, to the first yield criterion. As observed, the solution from the third yield criterion behaves rather similarly to that from the FEM. In contrast, for α values greater than 8°, prediction of drawing tension with the first and the second yield criteria differ markedly from that with the FEM.

Minimum drawing tension generally occurs between 3.5° and 14° (Avitzur, 1997). The solutions with the third yield criterion and with the FEM both have a point of minimum drawing tension within the range of 8–14°. The former can be thus used to determine the die angle that which minimises the drawing tension, as the prediction in said range is good.

Fig. 5 shows the percent error between the drawing tension values obtained with the third yield criterion and with the FEM, for two different friction coefficients μ. As observed in both cases, at die angles less than 17°, the error is less than 5%.

Fig. 6 shows the dimensionless drawing tension in function of the die angle for axisymmetric geometry, whereby the dimensionless drawing tension is defined as the drawing tension divided by the yield limit for the tension. As observed in the figure, for r values of 0.2, 0.3 and 0.4, the results obtained with the third yield criterion are much closer to the experimental data reported by Wistreich (1955) than are those obtained with the first yield criterion (the classical slab method). Thus the prediction with the third yield criterion is better than that made with the first yield criterion, for which the error increases with the die angle.

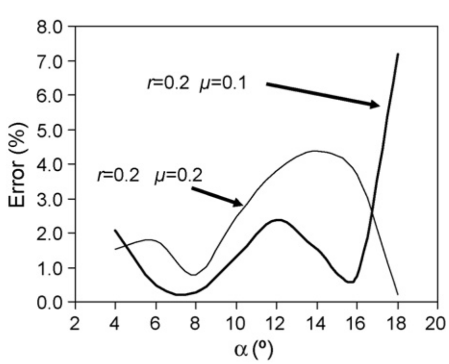

Fig. 5 – Percent error in the dimensionless drawing tension for values obtained from the FEM and from the third yield criterion, for symmetric dies.

Fig. 7 shows plots of percent error versus die angle for the three aforementioned reductions. The error shown in the graph is calculated as the difference in absolute value of the data predicted with the third yield criterion, and the experimental data of Weistrich, divided by the experimental values. For the three reductions studied, at die angles between 2.3° and 11.5°, the error is less than 9%.

5. Conclusions

We have developed a new model for predicting drawing tension. It is an analytical iterative model that offers straightforward numerical implementation. Notably, it incorporates the shear tension in the von Mises criterion, thereby improving upon conventional predictions of drawing tension.

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