Pergamon

Int. J. Mech. Sci. Vol. 39, No. 2, pp. 233–243, 1997

0020–7403/97 $15.00 + 0.00

PII: S0020-7403(96)00057-4

THEORETICAL METHOD FOR ESTIMATION OF MEAN PRESSURE ON CONTACT AREA BETWEEN ROLLING TOOLS AND WORKPIECE IN CROSS WEDGE ROLLING PROCESSES

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(*Received* 15 *December* 1995)

Abstract—The paper gives a new method no determine the mean unit contact pressures on a material–tool contact surface in cross wedge rolling processes (CWR). The dependencies worked out on the basis of the energy and the upper bound methods permits rolling forces to be determined which are comparable to experimentally measured ones. The analysis provides equations which relate the mean contact pressure q_m to the basic process parameters, namely the forming angle α, the spreading angle β, the relative reduction of a portion δ and the shear friction factors m and m_k. Copyright © 1996 Elsevier Science Ltd.

Keywords: cross wedge rolling, theoretical analysis, energy method, upper bound method.

NOTATION

A_{xy}, A_{xz}	projected contact areas in the radial and axial directions
L	length of wedge
Q	force
Q_x, Q_y, Q_z	tangential, axial and radial components of rolling force
R	roll radius
V	velocity
W_p	deformation work
W_t	friction work
W_z	external work
c	relative rolling pitch
d	forging diameter after rolling or reducing
d_0	billet diameter
d_z	substitute diameter of a rod
k	shear yield stress
l	rolling length, width of a sizing belt
l_z	substitute width of a sizing belt
m	shear friction factor on a forming area
m_k	shear friction factor on a sizing area
r	radius
r_0	billet radius
q_m	mean contact pressure
q_c	drawing stress
$\{r, z, v\}$	cylindrical co-ordinates
α	forming angle, cone semi-angle
β	spreading angle
δ	reduction of a portion
γ	ramp angle
λ	rolling coefficient
μ	friction coefficient
σ_0	yield stress
τ	shear stress
Δr	rolling depth

INTRODUCTION

The development of better and more advanced material forming processes aimed at high quality products at lower material and energy consumption, is a continuing area of research. It is particularly visible in metal forming processes, among which the modern technology of cross wedge rolling (CWR) has attracted attention.

The CWR method, developed intensively since the early 1960s, can be characterized by large capacity (limited only by the capacity of a heating device) and high profitability of the process. Products of such rolling processes are characterised by high precision (allowed tolerance of a diameter is 0.4 mm), good surface quality (obtained roughness $R_a = 5\mu m$) and higher strength of materials (fatigue strength rises 1.5–2 times and wear resistance by 20–40%) [1]. Material savings following this technology are from 20 to 60% in comparison to the other methods of axially-symmetric treatment such as open die forging, closed die forging, longitudinal forge rolling or machining [2]. At the level of applied technology, this method has mainly been used in the production of axially-symmetric forgings of a stepped shaft type and also preforms which are then press forged.

Despite the intense development of the process, many phenomena occurring during the CWR process have not yet been modelled theoretically. In particular, such features as the kinematics of material flow and the problems of the increase of product diameters, the determination of material–tool contact extent and conditions, the influence of technological conditions on the forming process stability and the problem of the formation of central cavities need attention. Because of the complexities associated with the deformation, theoretical analyses of the CWR process tend to employ only approximate solutions which are based on many simplifying assumptions.

The objective of this paper is to present a simplified method to determine the mean contact pressure between a tool segment and a forging in CWR processes.

CHARACTERISTICS OF CWR PROCESSES

Cross wedge rolling is a process of metal forming where a product (a forging or a preform) is worked as the result of the operation of tool wedge segments located on rolls or concave or flat penals on rolling mills (see Fig. 1). A typical tool segment for a cross wedge rolling process (Fig. 2) consists of four basic zones: a knifing zone, a guiding zone, a forming zone and a sizing zone. In the knifing zone, a tool enters a billet (to a depth Δr) reducing its diameter to the desired value d. Then over the length of the guiding zone this reduction advances to the whole perimeter of the product. This zone, especially in the cases of rolling at small Δr, is often neglected. In the forming zone, as the result of the reaction of side walls of the wedge the previously formed keyseat is widened in the direction from the centre towards the face surface of the product to the desired width l (or $2l$). The cross-sectional reduction is accompanied by free axial extension. Finally, over the length of the sizing zone the forging is subjected to pressure over the full dimension of the workpiece and those unwanted curvatures and other shape irregularities of the rolled product, formed during the first phase, are removed. Behind the sizing zone, there are side cutters which separate the deformed

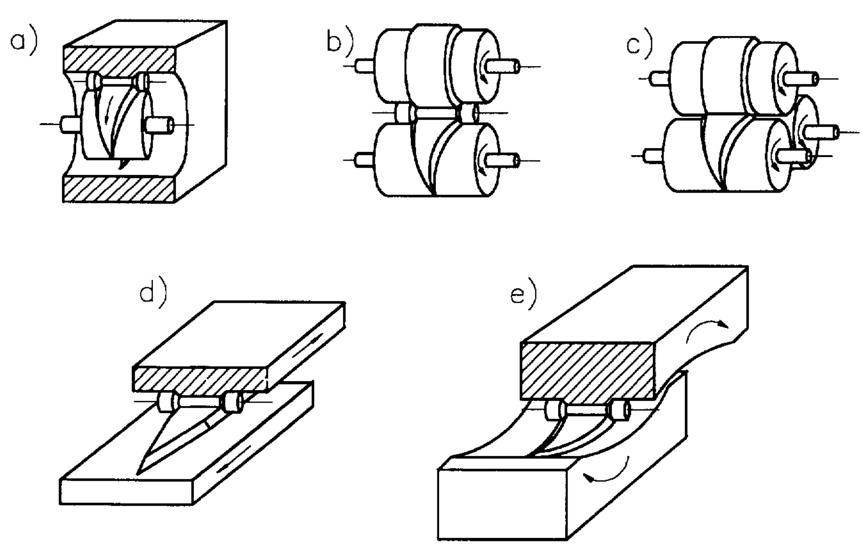

Fig. 1. Tooling configurations in a CWR: (a) one-roll; (b) two-roll; (c) three-roll; (d) flat wedge; (e) concave wedge.

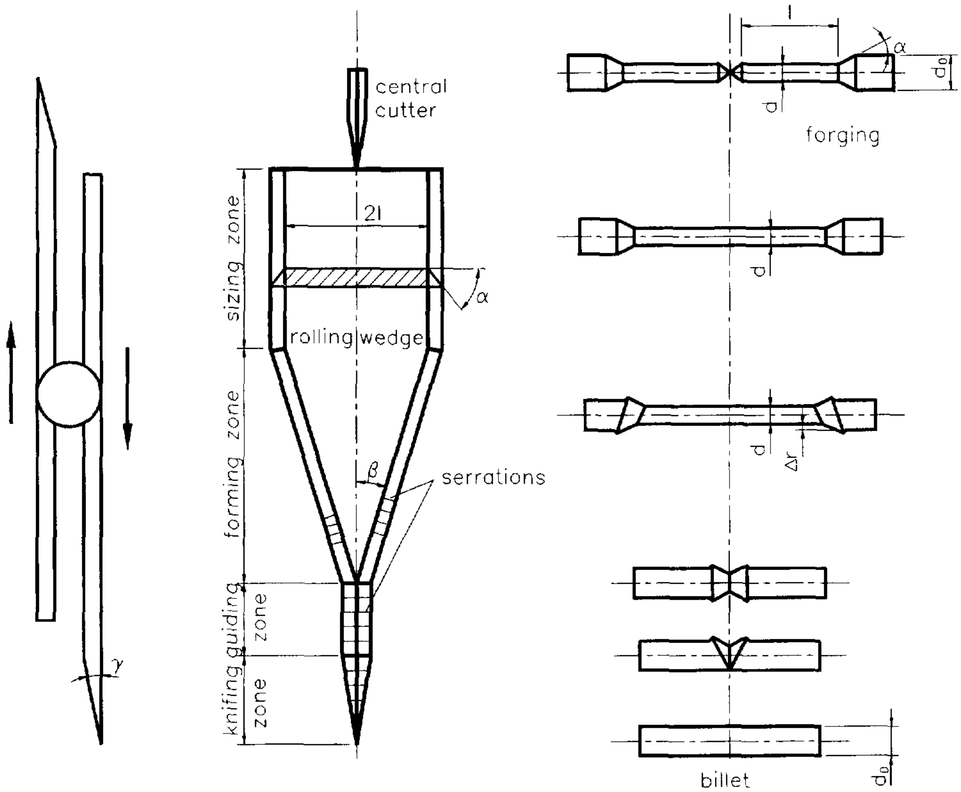

Fig. 2. Scheme of a CWR in the double system.

forging edges from the product or a central cutter to divide products in the case of rolling in a double system (Fig. 2).

The process of rolling in CWR processes depends mainly on the geometry of the tool segment used. This geometry is first and foremost characterised by a forming angle α, a stretching angle β, a ramp angle γ, the rolling depth Δr, the rolling length l and the tool segment length L. The forming deformation characterising the cross wedge rolling process is defined by the reduction of a portion δ, defined as follows:

$$\delta = \frac{d_0}{d}. \tag{1}$$

DETERMINATION OF CONTACT PRESSURE

It is essential to determine forces and rolling torque values for CWR processes. This makes possible not only the selection of suitable aggregates and proper devices for the process but also provides the basis to work out an automatic process control system. The rolling force Q in CWR processes can be determined when its components in the directions of the co-ordinate axes shown in Fig. 3 are known. These are the radial, Q_z, axial Q_y and tangential Q_x components. These particular components of the Q force can be presented as products of the metal–tool contact areas on projected onto relevant co-ordinate system planes and a mean unit pressure on the contact area.

Because of the complexity of the deformation zone geometry and the deformation mechanism, the theoretical analysis of the CWR process is difficult. That is why most publications which determine rolling forces employ experimental methods to determine mean unit pressures. Another simplification (mainly used by Hayama [3, 4]) is the assumption that unit pressures on the metal–tool contact area in CWR processes are equal to the pressures in cross rolling processes or in the side-pressing of cylinders. Among the references to work on CWR (two by Szczukin and Dubien [5] and Weroński and Pater [6]) provide the best guidance on analysing the process. These solutions apply the upper bound method to determine the unit pressure but [5] assume that the main directions of the metal flow are radial and tangential whilst Ref. [6] emphasizes radial and axial flow. The results obtained

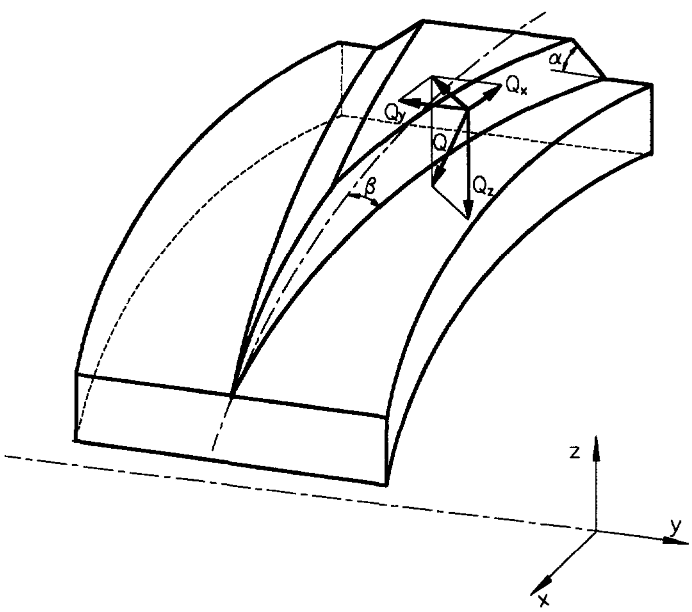

Fig. 3. Distribution of the force influencing a tool segment in CWR processes.

matched well the measured values. Unfortunately the complex mathematical tools applied in these solutions (the solutions were the results of multi-criterion optimization) do not allow these methods for estimating contact pressure values to be used extensively in industrial practice.

To work out the dependencies between the various parameters which allows one, relatively easily, to determine the mean unit pressure on the contact area, the analogy between the CWR and other reduction processes have been employed. Some previous work on wedge rolling following this approach can be found in Refs [7–9]. These solutions have been obtained on the basis of various engineering methods of analysis, such as energy methods and the upper-bound method. In Ref. [10] the author made an attempt to use the slab method to determine q_m, but the verification revealed that the results obtained through this method were too small.

The analysis carried out here assumes that the mean unit pressure in the CWR process is equal to the pressure in the reduction (extrusion) process of a round bar from a substitute diameter d_z to the diameter d (equal to the diameter of the product after rolling) (see Fig. 4). Thus we have the notion of the substitute reduction of a portion $\delta_z = d_z/d$.

The energy method

Because of the axial symmetry, the process of reduction through a conical die (Fig. 5) can be considered in the cylindrical co-ordinates $\{r, z, v\}$. The work performed by the external force Q_0 over the distance Δz_1 is given by:

$$W_z = Q_0 \Delta z_1 = q_m \sin\alpha\pi \frac{d_z^2 - d^2}{4} \Delta z_1. \tag{2}$$

But the total work of plastic deformation, assuming that the Haar–Karman's postulate of equality of two principal stresses is fulfilled, can be described by the following expression:

$$W_p = \frac{\pi d_z^2}{2} \sigma_0 \ln\frac{d_z}{d} (\Delta z_1 - \Delta z_2). \tag{3}$$

The dissipated energy as the result of friction at the die cone surface (characterized by a frictional shear factor m) and the sizing surface (m_k) can be determined from the equation:

$$W_t = mk \frac{d_z^4 - d^4}{d_z^2} \frac{\pi}{8\sin\alpha\cos\alpha} + m_k k\pi d l_z \Delta z_1. \tag{4}$$

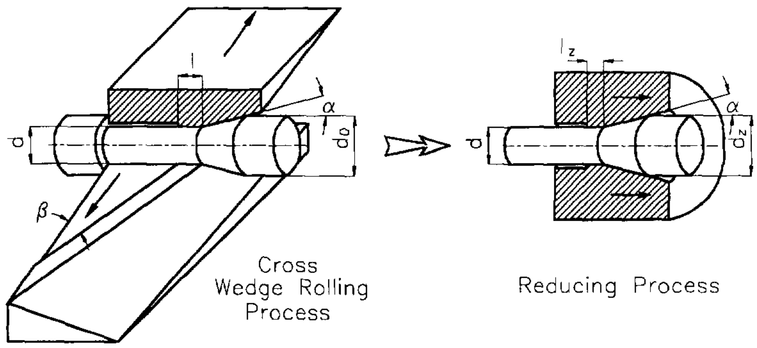

Fig. 4. Transformation of a CWR process into a rod reducing process.

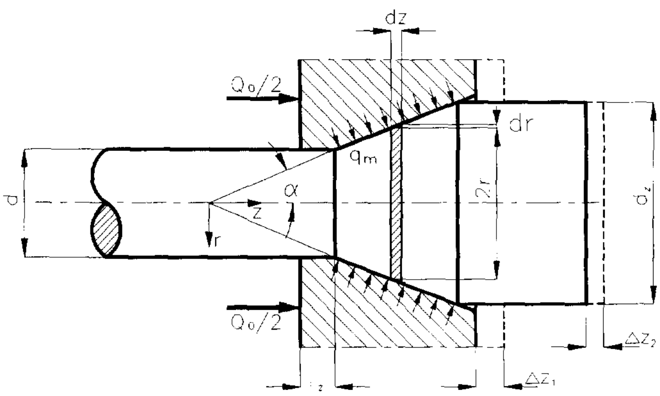

Fig. 5. Analysis of a rod reducing process using the energy method.

From the balance of the input energy (2) and the dissipated energy Eqns (3)–(4) we can obtain the dependence of the mean unit pressure in non-dimensional form:

$$\frac{q_{\mathrm{m}}}{\sigma_0} = \frac{2}{\sin\alpha\cdot(\delta_z^2 - 1)}\cdot\left(\ln\delta_z + \frac{2}{\sqrt{3}}\cdot m_{\mathrm{k}}\cdot\frac{l_z}{d}\right) + \frac{m}{\sqrt{3}\cdot\cos\alpha\cdot\sin 2\alpha}\cdot\frac{\delta_z^2 + 1}{\delta_z^2}. \tag{5}$$

The upper bound method

This solution is based on the assumption that the plastic deformation area is described by the so called spherical velocity field, described by Avitzur [11]. The material is assumed to deform plastically in the sub-area "B" which is in the form of a spherical sector bounded by the surfaces $r = r_1$ and $r = r_2$ and by the die cone surface, see Fig. 6(a). To obtain the solution, one transforms the system by adding velocity V_0 to each of its zones. Following the transformation, the reduction of a rod [Fig. 6(a)] is converted into the system presented in Fig. 6(b), characterizing a drawing process.

The drawing stress q_c of round bars determined by the upper bound method, in non-dimensional form, is expressed by [11]:

$$\frac{q_c}{\sigma_0} = \frac{2}{\sqrt{3}}\left[\left(\frac{f(\alpha)}{\sin^2\alpha} + m\cdot\mathrm{ctg}\,\alpha\right)\cdot\ln\frac{d_z}{d} + \frac{2\alpha - \sin\alpha}{2\sin^2\alpha} + 2m_k\frac{l_z}{d}\right], \tag{6}$$

where $f(\alpha)$ is determined by:

$$f(\alpha) = \frac{1}{2}\left[\sqrt{12} - \cos\alpha\sqrt{1 + 11\cos^2\alpha} + \frac{1}{\sqrt{11}}\ln\left(\frac{\sqrt{11} + \sqrt{12}}{\sqrt{11}\cos\alpha + \sqrt{1 + 11\cos^2\alpha}}\right)\right]. \tag{7}$$

Considering the external force balance of the formed object one can obtain the equation:

$$q_c\cdot\pi\cdot\frac{d^2}{4} = q_{\mathrm{m}}\cdot\pi\cdot\frac{d_z^2 - d^2}{4}. \tag{8}$$

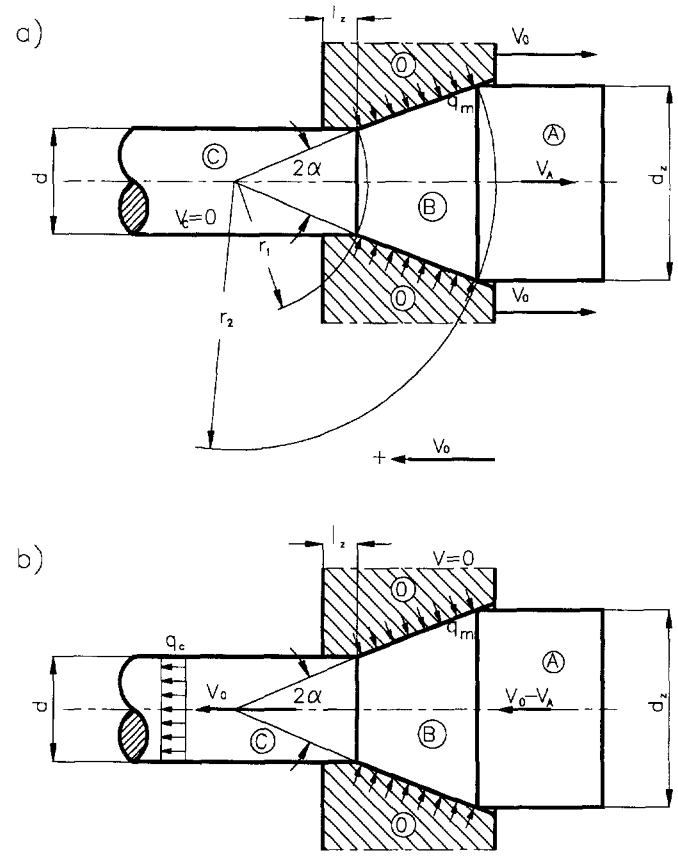

Fig. 6. Analysis of a rod reducing process using the upper bound method.

From the above one can determine the equation describing the mean unit pressures on the material–tool contact area in the analysed reduction process

$$\frac{q_m}{\sigma_0} = \frac{1}{\delta_z^2 - 1} \cdot \frac{2}{\sqrt{3}} \left[\left(\frac{f(\alpha)}{\sin^2 \alpha} + m \cdot \operatorname{ctg} \alpha \right) \ln \delta_z + \frac{2\alpha - \sin 2\alpha}{2 \sin^2 \alpha} + 2m_k \frac{l_z}{d} \right]. \tag{9}$$

Selecting a substitute diameter d_z and in the result a substitute δ_z needed some experimental verification aimed at measurements of the Q_z force during a rolling process.

The experiments were carried out in a laboratory for CWR equipped with tool segments of various geometry. Forces were measured with force transducers. An example of the variation of the radial force occurring during the CWR processes is presented in Fig. 7. It follows from this graph that the force increases steadily in the knifing zone, reaches its maximum and a relatively stable form in the stretching zone and then gently decreases in the sizing zone. Table 1 presents the results of some measurements of the maximum radial force Q_z, obtained at various CWR parameters.

To determine the substitute diameter d_z providing q_m values close to the real ones (measured) it was previously assumed that d_z is: (1) $d_z = d + 2\Delta r/3$; (2) $d_z = d + \Delta r$; (3) $d_z = d + 4\Delta r/3$; and (4) $d_z = d_0$ where Δr is the maximum rolling depth obtained in a CWR process ($\Delta r = (d_0 - d)/2$). Then the radial force Q_z in CWR processes was computed at $\alpha = 20°$ and $\beta = 9°$. This force was determined on the basis of the equation:

$$Q_z = 2(q_m \cdot A_{xy} - \tau \cdot \sin \beta \cdot A_{xz}), \tag{10}$$

where A_{xy} and A_{xz} were determined on the basis of the equations given in Table 2. The computations assume that the yield stress of lead σ_0 used in the experimental work is 17.2 MPa and that the width

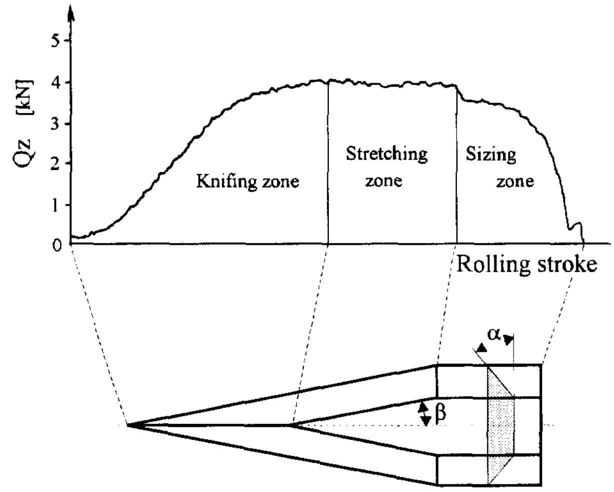

Fig. 7. Distribution of the radial force Q_z during the CWR process at $\alpha = 40°$, $\beta = 3°$, $\delta = 1.43$, $d_0 = 20$ mm; sample material: lead.

Table 1. Some results of measurements of the maximal radial force during rolling with flat wedges; sample material: lead

Test no.	α (°)	β (°)	δ	r_0 (mm)	Q_z (N)	Q_z/r_0^2 (N/mm²)
1	20	9	1.29	9.0	5367	66.26
2	20	9	1.40	7.0	3599	73.45
3	20	9	1.50	9.0	5493	67.82
4	20	9	1..67	10.0	6567	65.67
5	20	9	1.67	10.0	6630	66.30
6	20	9	1.80	9.0	4925	60.80
7	30	9	1.29	9.0	4609	56.90
8	30	9	1.43	9.0	4736	58.46
9	30	9	1.46	8.4	3915	55.48
10	30	9	1.50	9.0	4357	53.79
11	30	9	1.56	9.0	4167	51.45
12	40	3	1.27	7.0	1957	39.94
13	40	3	1.40	7.0	2210	45.10
14	40	3	1.43	10.0	4104	41.04
15	40	3	1.50	9.0	3283	40.53
16	40	6	1.43	10.0	4167	41.67
17	40	9	1.50	9.0	4167	51.44
18	50	9	1.50	9.0	3662	45.21

of the substitute sizing distance l_z is equal to half of the rolling pitch divided by the relative reduction of a portion δ.

The results of the computations are graphs of the radial force Q_z divided by r_0^2 vs δ depending on the assumed value of the substitute diameter d_z. These graphs are plotted on the basis of the energy method in Fig. 8 and of the upper bound method in Fig. 9. The results obtained in the experiments are also indicated in these figures. The comparison between the measured values and the ones obtained theoretically indicates that the best conformity between the results is obtained by assuming $d_z = d + \Delta r$. The results of theoretical computations and experimental measurements for the CWR processes for different parameters are given together in Figs 10 and 11 which demonstrate the correctness of the solution. These results show the validity of the method applied to determine the radial force Q_z and subsequently the mean unit contact pressure. Moreover, it should be stressed that the upper bound method results in slight overrating and the energy method leads to a certain underrating of the radial force (unit pressure) as one would expect. The dependence of the radial

Table 2. Formulas for determination of contact area projections on xy and xz planes of coordinate system [1]

Variant of CWR	Formulae
$c \leqslant 1$	$A_{xy} = \frac{2}{3} r_0^2 \frac{\cos\beta}{\mathrm{tg}\,\alpha} \sqrt{\frac{3}{1 \pm \frac{r_0}{R}} \cdot \frac{\delta - 1}{\delta}} \left[1 + c\frac{\delta - 1}{\delta}\left(1 + \sqrt{\frac{2 + c\delta - c}{2\delta}}\right) - \sqrt{\left(\frac{1 + c\delta - c}{\delta}\right)^3} \right]$ $A_{xz} = \frac{2}{3} \cos\beta r_0^2 \sqrt{\frac{3}{1 \pm \frac{r_0}{R}} c \frac{\delta - 1}{\delta}} \cdot \left[1 - \sqrt{\left(\frac{1 + c\delta - c}{\delta}\right)^3} + c\frac{\delta - 1}{\delta} \right]$
$c > 1$	$A_{xy} = \frac{2}{3} \cos\beta r_0^2 \frac{\cos\beta}{\mathrm{tg}\,\alpha} \frac{\delta - 1}{\delta} \sqrt{\frac{3}{1 \pm \frac{r_0}{R}}} \cdot \left[1 + \frac{3}{2}(c - 1) + \sqrt{\frac{\delta + 1}{2\delta}} \right]$ $A_{xz} = \frac{2}{3} \cos\beta r_0^2 \frac{\delta - 1}{\delta} \sqrt{\frac{3}{1 \pm \frac{r_0}{R}} \cdot \frac{\delta - 1}{\delta}}$

where $c = \frac{\pi \cdot \mathrm{tg}\,\beta \cdot \mathrm{tg}\,\alpha \cdot \lambda \cdot \delta}{\delta - 1}$ and λ is defined by Eqn (15).

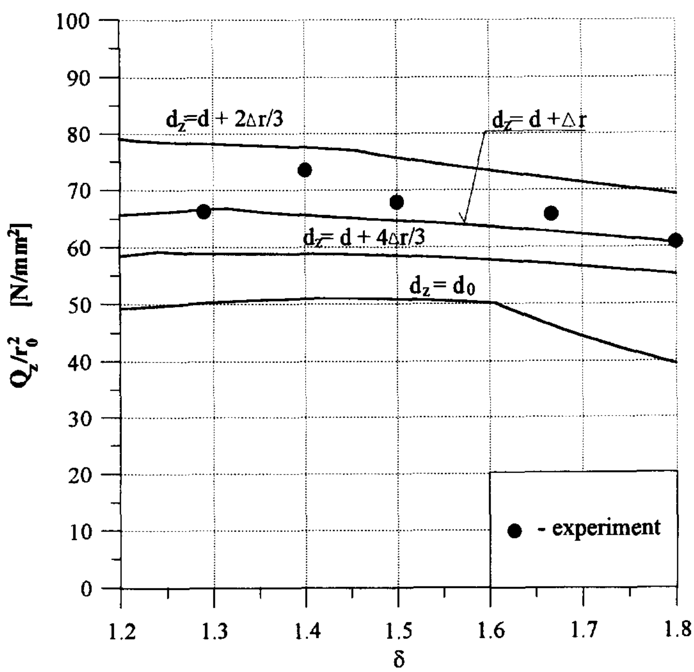

Fig. 8. Dependence of Q_z/r_0^2 on the relative reduction of a portion determined by the energy method for the CWR process at $\alpha = 20°$, $\beta = 9°$, $\mu = 0.30$ and $\mu_k = 0.18$.

force Q_z in relation to r_0^2 vs the relative reduction of a portion δ is interesting. Despite the increase in δ which is followed by an increase in the contact surface area, this force tends to reduce. This form of the radial force vs δ variation is confirmed by the experimental investigations carried by Awano and Danno [12].

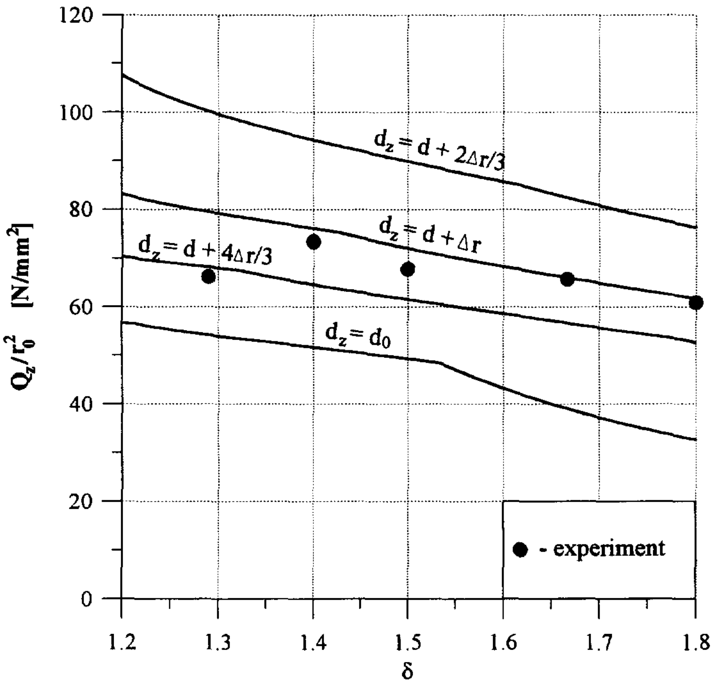

Fig. 9. Dependence of Q_z/r_0^2 on the relative reduction of a portion determined by the upper bound method for the CWR process at $\alpha = 20°$, $\beta = 9°$, $\mu = 0.30$ and $\mu_k = 0.18$.

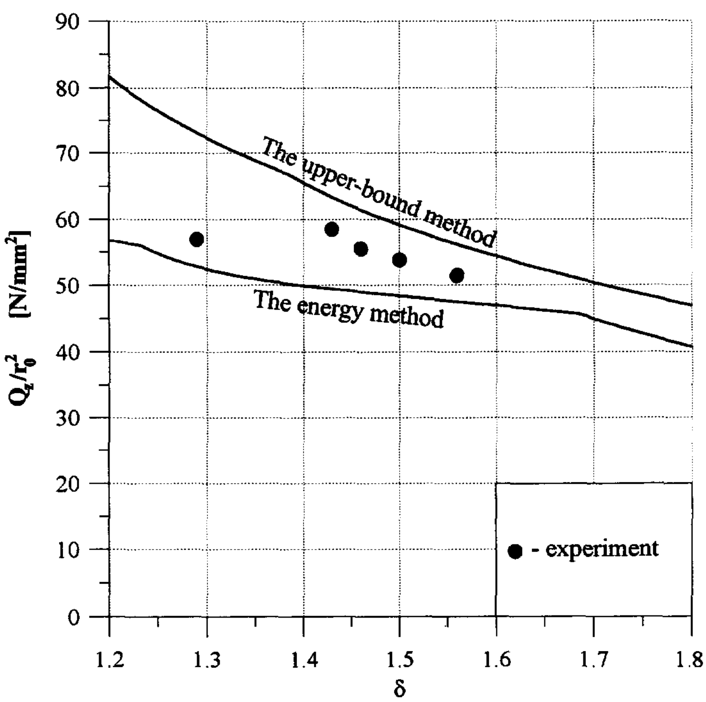

Fig. 10. Dependence of Q_z/r_0^2 on the relative reduction of a portion determined for the CWR process at $\alpha = 30°$, $\beta = 9°$, $\mu = 0.30$ and $\mu_k = 0.18$.

Summarizing, on the basis of the considerations above, Eqns (5) and (9) are recommended for determining unit pressure. Considering that

$$\delta_z = \frac{d + \Delta r}{d} = \frac{d_0 + d}{2d} = \frac{\delta + 1}{2}, \tag{11}$$

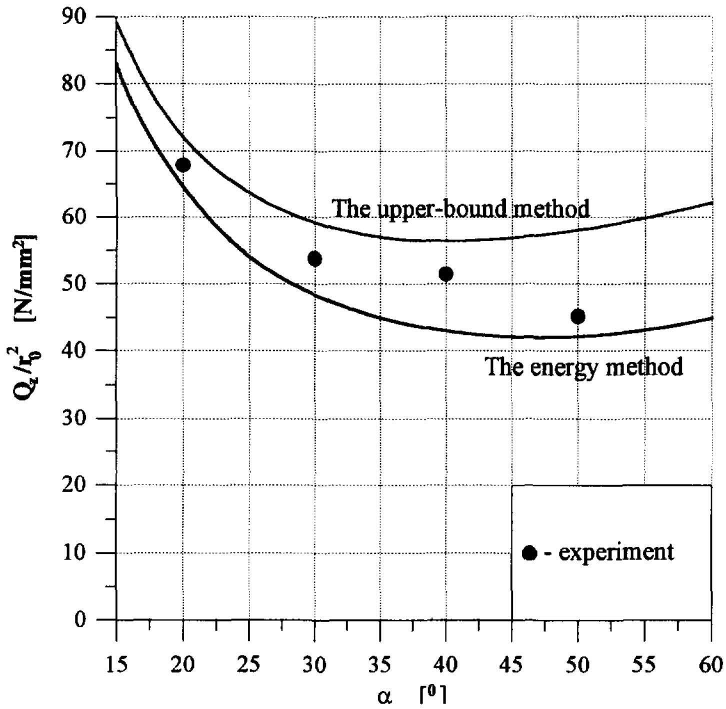

Fig. 11. Dependence of Q_z/r_0^2 on the forming angle α determined for the CWR process at $\beta = 9°$, $\delta = 1.5$, $\mu = 0.30$ and $\mu_k = 0.18$.

and

$$l_z = \frac{\pi r_0 \lambda \,\mathrm{tg}\beta}{2\delta}, \tag{12}$$

we can obtain the following final forms:

(a) the energy method

$$\frac{q_m}{\sigma_0} = \frac{8}{(\delta+1)^2 - 4}\cdot\left[\ln\left(\frac{\delta+1}{2}\right) + \frac{m_k}{2\sqrt{3}}\pi\lambda\,\mathrm{tg}\beta\right] + \frac{m}{\sqrt{3}\sin 2\alpha}\cdot\frac{(\delta+1)^2+4}{(\delta+1)^2}; \text{ and} \tag{13}$$

(b) the upper bound method

$$\frac{q_m}{\sigma_0} = \frac{8}{\sqrt{3}[(\delta+1)^2-4]}\cdot\left[\left(\frac{f(\alpha)}{\sin^2\alpha} + m\,\mathrm{ctg}\alpha\right)\cdot\ln\left(\frac{\delta+1}{2}\right) + \frac{2\alpha - \sin 2\alpha}{2\sin^2\alpha} + \frac{m_k}{2}\pi\lambda\,\mathrm{tg}\beta\right], \tag{14}$$

where the rolling coefficient λ is defined by the following equation as a function of the basic rolling parameters (α, β and δ) [1]:

$$\lambda = (2.587 - 1.557\cdot\delta^{0.3528})\cdot(0.00355\cdot\alpha + 0.927)\cdot\beta^{0.0568}. \tag{15}$$

A special computational procedure has been employed to correlate the frictional shear factor m and friction coefficient μ. This procedure uses definitions of the constant friction model and the Coulomb friction model. The results are given in the form of correlating nomograms of μ and m in Ref. [1].

CONCLUSION

The paper presents a new simplified method to determine contact pressure dependent on the basic process parameters (α, β, δ, m and m_k). The given method to determine q_m by means of the energy or the upper bound methods makes it possible to obtain results comparable to measured ones. The account above demonstrates the suitability of the method described for deriving results for the CWR process. This method can also be employed to determine the limiting values of the basic parameters of the CWR process at which it is stable, without necking of the rolled product and without a slip.

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