

A Comparative Study of Various Flow Instability Criteria in Processing Map

Ma Xiong¹, Zeng Weidong¹, Sun Yu¹, Zhao Yongqing², Wang Shaoli¹,
Zhou Yigang¹

¹ State Key Laboratory of Solidification Processing, Northwestern Polytechnical University, Xi'an 710072, China; ² Northwest Institute for Nonferrous Metal Research, Xi'an 710016, China

Abstract: Four instability criteria, namely Murty, Gegel, Malas and Prasad criteria, were compared, and the physical significance of parameters was analyzed in this paper. It is indicated that the instability map developed by Murty criterion is similar to Prasad criterion, showing an unstable flow at high strain rates. Murty criterion exhibits a little narrower unstable region. The instability maps developed by Malas criterion and Gegel criterion have similar shapes, and the unstable regions are wider than those Prasad criterion and Murty criterion. In addition to the unstable flow at high strain rates as predicted by Murty criterion and Prasad criterion, Malas criterion and Gegel criterion have a good ability for predicting unstable flow at high temperatures and low strain rates. The processing maps were validated by hot compression tests of Ti-22Al-25Nb alloy, and the possible causes of various instability were discussed.

Key words: processing map; instability criterion; hot compression tests; Ti-22Al-25Nb alloy

Processing map was first proposed by Raj in 1981^[1], and then developed by Prasad et al based on dynamic material modeling (DMM). In recent years, processing maps have been developed for use in optimizing hot workability and controlling the microstructure of the product.

The plastic instability criteria is very important for determining the “safe” or “unsafe” regions during hot working procedures, which has received considerable attentions all over the world. In the past 20 years, researchers have proposed many plastic instability criteria^[2-9]. Among them, Prasad’s criteria was the most popular instability criteria and has been validated in the titanium alloys, high-temperature alloys, aluminum alloys and composite materials^[2]. However, Prasad et al assumed the strain rate sensitivity parameter (m) to be a constant, which was questioned by some scholars. For this reason, Murty et al^[3,4] proposed a more strict plastic instability criteria in which m is a variable. Gegel^[5] derived a novel stability criterion based on the second law of

thermodynamics theory and Liapunov stability equation. Malas^[6] put forward an idea that m replaces η in Gegel criterion to build another group of stability criteria. Semiatin^[7] and Montheillet^[8] et al. suggested that plastic instability behavior was related to the work- hardening rate and the strain rate sensitivity parameter, and then established a phenomenological criteria using test methods. Chen^[9] established the plastic criteria using the stress ratio.

It is shown that the existing criteria for determining the “safe” and “unsafe” regions during hot working procedures are different in their theoretical basis, formula and physical significance, sometimes even contradictory. Some review articles about the plastic instability criteria can be found in many literatures^[10-13]. However, a comparative study of various plastic instability criteria for processing map has not been reported yet.

In the present study, a comparison of four plastic instability criteria (including Prasad, Murty, Gegel and Malas) was

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Corresponding author: Ma Xiong, Candidate for Ph. D., State Key Laboratory of Solidification Processing, Northwestern Polytechnical University, Xi'an 710072, P. R. China, Tel: 0086-29-88494298, E-mail: mxnwp@163.com; Zeng Weidong, Professor, E-mail: zengwd@nwpu.edu.cn

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carried out based on the hot compression experiment data of Ti-22Al-25Nb alloy^[14,15].

1 Plastic Instability Criteria

One objective of hot working is to avoid defects and plastic instability phenomena, such as adiabatic shear bands, local plastic flow, void formation, cracking etc. Therefore, the researchers have proposed various kinds of plastic instability criteria^[10-13] as follows.

1.1 Prasad's instability criterion

Prasad^[2] has developed a criterion for evaluating the flow instabilities. The criterion is based on the extremum principles of irreversible thermodynamics as applied to large plastic flow, and the flow instability will occur during hot deformation if

$$\xi(\dot{\epsilon}) = \frac{\partial \ln \left(\frac{m}{m+1} \right)}{\partial \ln \dot{\epsilon}} + m < 0 \quad (1)$$

where m is the strain rate sensitivity of flow stress, $\dot{\epsilon}$ is strain rate, and $\xi(\dot{\epsilon})$ is the instability parameter.

1.2 Gegel's stability criteria

Gegel's approach^[5] is based on the Liapunov stability criteria, and derived the following four inequations for stable material flow:

$$\begin{cases} \frac{\partial \eta}{\partial \ln \dot{\epsilon}} < 0 \\ \frac{\partial s}{\partial \ln \dot{\epsilon}} < 0 \end{cases} \quad \begin{matrix} 0 < m < 1 \\ s > 1 \end{matrix} \quad (2)$$

here ,

$$s = \frac{1}{T} \frac{\partial(\ln \sigma)}{\partial(1/T)} \quad (3)$$

where η is the efficiency of power dissipation, s is the temperature sensitivity, σ is the flow stress, and T is the temperature.

It should be pointed out that the Prasad's criterion and Gegel's instability criterion were based on the assumption that the strain rate sensitivity parameter (m) in $\sigma = C\dot{\epsilon}^m$ is independent of strain rate.

1.3 Murty's instability criterion

Murty^[3,4] considered that strain rate sensitivity parameter m is not a constant, so the expression of power dissipation efficiency η was derived in terms of the definition of J co-content:

$$\eta = \frac{J}{J_{\max}} = 2 \left(1 - \frac{1}{\alpha \dot{\epsilon}} \int \sigma d\dot{\epsilon} \right) \quad (4)$$

where J_{\max} is the maximum of co-content.

Thus, for stable material flow:

$$2m < \eta \text{ or } \eta > 0 \quad (5)$$

The instability criterion (5) is valid for any type of $\sigma-\dot{\epsilon}$ curve. Murty's criterion has a strict theory basis and is simple,

but it is difficult to take the calculation of integration.

1.4 Malas's stability criterion

Malas^[6] considered that η is the same as m . By substituting m in Gegel's criterion with η , they proposed the following four conditions for the stable material flow:

$$\begin{cases} \frac{\partial m}{\partial \ln \dot{\epsilon}} < 0 \\ \frac{\partial s}{\partial \ln \dot{\epsilon}} < 0 \end{cases} \quad \begin{matrix} 0 < m < 1 \\ s > 1 \end{matrix} \quad (6)$$

1.5 Phenomenological criterion

Semiatin's instability criteria^[16] describes the relation between the rate of flow softening and the strain rate sensitivity parameter through the parameter α for plane strain compression, which is based on the principle of force equilibrium:

$$\alpha = -\frac{\gamma}{m} \quad (7)$$

where α is an instability parameter.

On the basis of microstructural observations in titanium alloys, a limit on the workability parameter has been fixed for flow localization or fracture^[17]:

$$\alpha > 5 \quad (8)$$

1.6 Other criteria

Other plastic instability criteria, such as the stress ratio criteria^[9] and polar reciprocity model^[18] et al, are not introduced in detail for the brevity of the representation.

2 Test Data

The experimental data for generating processing maps in this paper were all from the hot compression tests of Ti-22Al-25Nb alloys in literatures^[14,15]. The test temperature is in the range of 940 to 1060 °C, and strain rate range of 0.01-10 s⁻¹. The data at strain of 0.7 is shown in Table 1.

The flow instability phenomena of Ti-22Al-25Nb alloy during the hot deformation in literature^[14] were summarized as follows:

Adiabatic shear bands and cracking along 45° occurred in temperature ranges of 940-970 °C and strain rate of 0.4-10 s⁻¹, while flow localization and longitudinal cracking occurred in the temperature ranges of 970-1060 °C and strain rate of 1-10 s⁻¹.

3 Results

3.1 Prasad's criterion

A processing map based on Prasad's instability criterion is shown in Fig.1. It can be seen that the efficiency of power

Table 1 Flow stress data of Ti-22Al-25Nb alloy at strain of 0.7

Strain	Strain rate, $\dot{\epsilon}$ /s ⁻¹	Temperature/				
		940	970	1000	1030	1060
0.7	0.01	150.9	114.1	71.7	42.6	25.6
	0.1	260.8	182.9	123.7	82.6	70.5
	1	425.5	275.1	227.6	155.9	142.7
	10	518.0	358.0	360.5	226.7	217.4

dissipation η increases with increasing of deformation temperature and decreasing of strain rate. The peak of η appears in the regime of high temperature and low strain rate, indicating a good workability.

Plastic instability occurs in the regime of low temperature and high strain rate (left upper) with an efficiency of power dissipation less than 25%, which is corresponding to adiabatic shear bands or cracking along 45°. In the regime of high temperature and high strain rate (right upper), the materials fall into the instability regime with longitudinal cracking or flow localization. Other regimes are safe^[14].

3.2 Murty's criterion

The processing map based on Murty's instability criterion is shown in Fig.2.

Although Murty's criterion has used a strict formula for solving the integral of η , the isolines distribution of the power dissipation efficiency are very similar to Prasad's, which suggest that the flow stress-strain curve of Ti-22Al-25Nb alloy obeys a power law in the experiment. For the same reason, the plastic instability regimes of the two criteria have a similar shape, and instability map by Prasad's criterion is slightly wider than that by Murty's one.

3.3 Gegel's criterion

The contour map of the power dissipation efficiency based on Gegel's criterion is the same as Prasad's criterion, so it is not shown repeatedly here. The plastic instability conditions

are indicated in Eq. (2).

The instability map defined by $\partial\eta/\partial\ln\dot{\epsilon}$ is shown in Fig.3. If η decreased with $\ln\dot{\epsilon}$ at any strains and temperatures, i.e. $\partial\eta/\partial\ln\dot{\epsilon} < 0$, the plastic flow is unstable. The predicted instability regions are in the temperature range of 1000-1050 °C and strain rate range of 0.01-0.1 s⁻¹ (lower right corner) with high values of η , which was not predicted by Prasad's and Murty's criteria. The flow instability with such high efficiency of power dissipation has been observed by Venugopal^[19] and Prasad^[2] in AISI 316L stainless steel and titanium alloys. It is proved to be related to wedge cracking caused by grain boundary sliding. However, further analysis is required to confirm these mechanisms in our study.

Flow instability of $m < 0$ is shown in Fig.4 with a shadow regime. Based on the dynamic material model, it is effective to evaluate the workability of materials by m , because the strain rate sensitivity parameter reflects the distribution between G content and J co-content in total power. The tendency of flow instability decreased with increasing of m . If $m < 0$, the flow instability such as dynamic strain aging or micro-cracking may occur. In this study the flow instability may be attributed to the adiabatic shear bands and shear cracking along 45°^[12].

The contour map of temperature sensitive parameter (s) is shown in Fig.5. The instability regimes are defined by $s < 1$, which is in the temperature range of 960-1000 °C and at strain rate (>1 s⁻¹), or at the temperature (>1040 °C) and strain rate

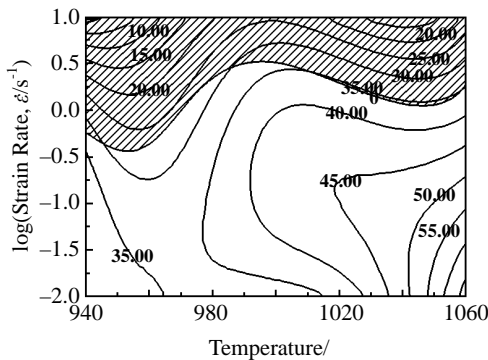


Fig.1 Processing map on Prasad's criteria

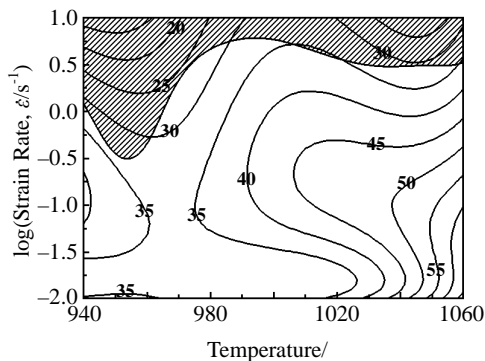


Fig.2 Processing map on Murty's criteria

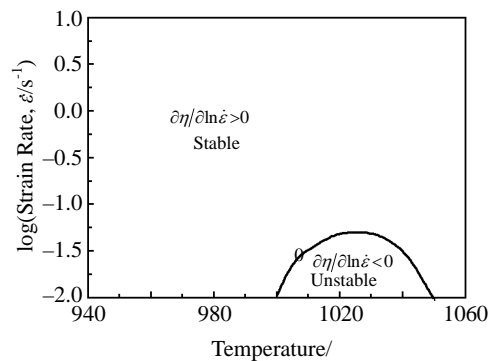


Fig.3 Instability map of η'

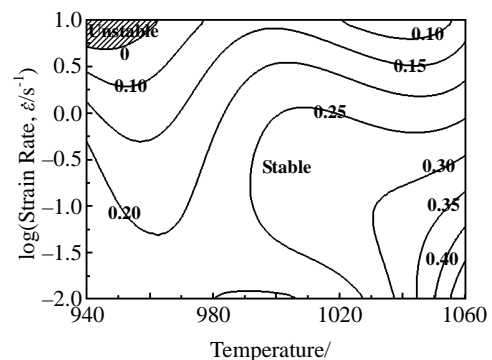


Fig.4 Contour map of m

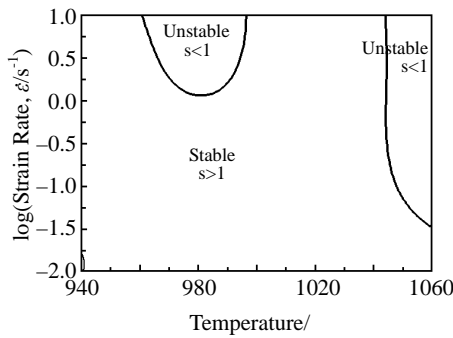


Fig.5 Contour map of s

($>10^{-1.5} \text{ s}^{-1}$). If $s>1$, it means that the rate of entropy production is always positive as required for an irreversible process, and the flow is stable.

The instability map defined by the parameter of $s' = \partial s / \partial \ln \dot{\epsilon}$ is shown in Fig.6. If $s'>0$, the flow instability occurs at low temperature ($<960 \text{ }^\circ\text{C}$) and the strain rate ($10^{-1.7} - 10^{0.7} \text{ s}^{-1}$), or at the high temperature (1000-1040 $^\circ\text{C}$) and high strain rate ($>1 \text{ s}^{-1}$). s' is related to flow instability as flow localization caused by adiabatic shear effect. $s'>0$ indicates a positive dependence of temperatures on flow stresses with the increasing of strain rates; temperature rise in the adiabatic shear bands will be more and hence unstable.

The processing map with Gegel's instability criterion is shown in Fig.7. It can be seen that the instability regions determined by Gegel's instability criterion are much wider than Prasad's criterion or Murty's criterion. The flow instability occurs in the high strain rate region (upside), low temperature and low strain rate region (left side), and high temperature and low strain rate region (right corner).

3.4 Malas's criterion

Malas's criteria is almost the same as Gegel's criterion except for the instability condition of $\partial m / \partial \ln \dot{\epsilon} < 0$, which is shown in Fig.8. The other conditions are the same as Gegel's criterion.

The processing map determined by Malas's criterion in Fig.9 is similar to Fig.7. The results indicate that the instability regimes based on Gegel's criterion and Malas's criterion in Ti-22Al-25Nb alloy are the same.

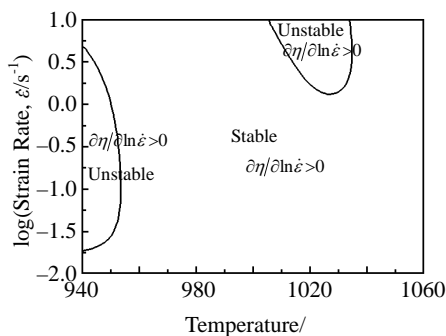


Fig.6 Instability map of s'

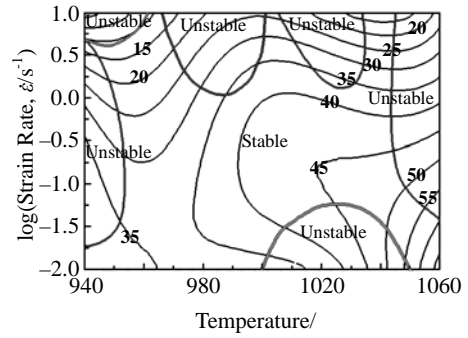


Fig.7 Processing maps on Gegel's criteria

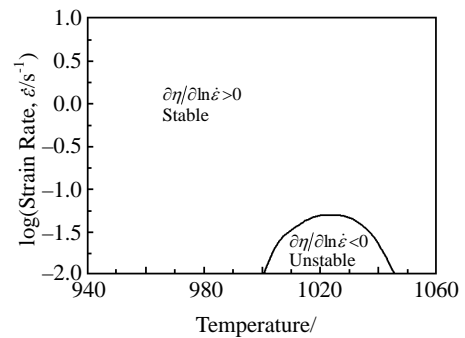


Fig.8 Instability map of m'

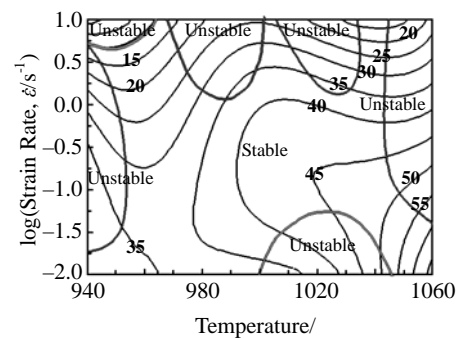


Fig.9 Processing map on Malas's criterion

4 Discussion

4.1 Comparison of Prasad's criterion and Murty's criterion

Although the two instability criteria are very different from each other in expressions, the processing maps obtained by the two instability criteria are similar in shape. The flow instability all occurs at high strain rate, which is consistent with the instability phenomena such as adiabatic shear bands and 45° shear cracking at low temperature and high strain rate, or flow localization and longitudinal cracking at high temperature for the Ti-22Al-25Nb alloy. It indicates that the two instability criteria are more accurate for predicting the flow instability at high strain rate.

The instability region predicted by Murty's criterion is slightly narrower than that by Prasad, which is due to the

different treatments on the strain rate sensitivity parameter m . The Prasad's instability criterion is derived based on the assumption that m is a constant and the flow stresses follow the equation of $\sigma=K\cdot\dot{\epsilon}^m$. Murty's instability criterion is derived by the definition of J co-content, in which m is a variable, and can be applied to any type of stress-strain curve. Strictly speaking, Eq. (1) is valid only when m is a constant; if m is not a constant and dependent on strain rate, Eq.(1) is wrong. Murty's criterion of Eq. (4) is independent on m .

In general, Prasad's instability criterion should be selected to construct processing maps to avoid complicated calculations. However, Murty's criterion should be selected for more accurate predictions because of its strict theoretical inferences and calculations.

4.2 Comparison of Gegel's criterion and Malas's criterion

Processing maps based on Gegel's criterion and Malas' criterion are almost the same except one flow instability region occurring at high temperature and low strain rate (right corner). Gegel's criterion and Malas' criterion can predict the flow instability at high strain rate as Prasad's criterion and Murty's criterion, showing a wider instability region and are much safer for the control of hot working procedures.

In Gegel's criterion, the efficiency of power dissipation η was used to evaluate the instability region, which reflects the proportion of the energy dissipation with respect to the linear dissipation energy. While in Malas's criterion, the strain rate sensitivity parameter m was used to characterize the flow instability region, which shows the proportion of G content and J co-content in the total power. The efficiency of power dissipation η and the strain rate sensitivity parameter m present the proportion of the energy dissipation with respect to the total energy during the hot working procedures, so it is not surprising that the flow instability determined by the two criteria are basically identical. Gegel's criterion is derived based on the assumption that m in equation $\sigma=K\cdot\dot{\epsilon}^m$ is a constant, while Malas' criterion does not care whether the m is a constant or not, so Malas's criterion is more reasonable.

4.3 Comparison of the four criteria

Compared with Prasad's criterion and Murty's criterion, the instability regions predicted by Gegel's criterion and Malas's criterion are wider. Gegel's criterion and Malas's criterion can predict the flow instability at high temperature and low strain rate (right corner of Fig.7 and Fig.9), which is not predicted by Prasad's criterion and Murty's criterion under the test conditions. It may be related to the wedge cracking.

It can be seen from the processing map of Ti-22Al-25Nb alloy that all kinds of plastic instability criteria are not consistent completely, sometimes even contradictory. Therefore, these criteria should be chosen carefully and considered comprehensively in practical use.

Generally, for the pure metal, the low alloying materials and the materials which obey the power law, Prasad's criterion is the preferential choice for optimizing the hot workability,

which has been proven in many alloys. Murty's criterion should be selected for more accurate predictions because of its strict theoretical inferences and calculations. Gegel's criterion and Malas's criterion have a distinct physical significance and much wider instability regions predicted, so they are safer. However, the instability regions predicted by Gegel's criterion and Malas's criterion at high strain rate are discontinuous, so further study on accuracy of these criterion is required.

5 Conclusions

1) The instability regions predicted by Murty's criterion are similar to those predicted by Prasad's criterion, which is located in high strain rate region. Prasad's criterion has a wider unstable region.

2) The flow instability predicted by Gegel's criterion and Malas's criterion is similar, which occurs in high strain rate region or high temperature and low strain rate region.

3) The flow instability predicted by Prasad's criterion and Murty's criterion are more effective at high strain rate, which are the preferential choices in most cases. Gegel's criterion and Malas's criterion can predict the flow instability at high temperature and low strain rate. However, further study for validation by experiment is required.

4) Various criteria should be considered to optimize the parameters of hot workability.

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