Abstract

Smart material technology has become an area of increasing interest for the development of lighter and stronger structures, which are able to incorporate sensor and actuator capabilities for collocated control. However, to obtain the optimized control performance, the actuators must be placed at locations to excite the desired modes most effectively. In this work a plate with bonded piezoelectric elements will be modeled by Finite Element Method using MATLAB®. The approach proposed for the design consists in the determination of $H_{\infty}$ norm of each sensor/actuator candidate position for selected modes of the plate. The $H_{\infty}$ norm will be computed using linear matrix inequalities techniques. The paper concludes with an experimental application in a plate. The results were compared with those supplied by the ANSYS® program.

Keywords: Smart material, optimal placement of sensor and actuator, $H_{\infty}$ norm

INTRODUCTION

In recent years, active vibrational control of flexible structures has been the subject of intensive research. The majority of the research works on active vibrational control of flexible structures employs piezoelectric materials, such as sensors and actuators, because of some advantages when compared with conventional materials. These materials are composed by piezoelectric ceramic (PZT - Lead Zirconate Titanate), commonly used as distributed actuators, and piezoelectric plastic films (PVDF - PolyVinylidene Fluoride), highly indicated for distributed sensors, Clark et al. (1998).
Active vibration control encompasses three main phases: structural design, optimal placement of sensor and actuator and controller design. Models of interaction considering the electromechanical coupling between actuators and host structures have been presented by Crawley and de Luis (1987) and by Crawley and Anderson (1990). Several authors studied the problem of optimal placement of actuators in active structural control using different methodologies, Lopes Jr. et al. (2000). One of the more mentioned techniques in literature uses genetic algorithms (GA) as a procedure of optimization. Simpson and Hansen (1996) use a simple model of an interior aircraft to determine the optimal location of PZT. Furuya and Haftka (1993) find the optimal placement of 8 actuators in a structure with 1507 candidate position. Gabbert et al. (1997) present a technique based on classical methods to determine actuator placement in smart structures by discrete-continuous optimization. The approach proposed in the present article considers the determination of the $H_\infty$ norm for each sensor/actuator candidate position for selected modes of the system. The $H_\infty$ norm is computed by using linear matrix inequalities techniques (LMI).

**STRUCTURAL MODELING**

While piezoelectric elements exhibits nonlinear hysteresis at high excitation levels, the response required in current typical structural applications is approximately linear. In this work we will use the linear constitutive relations for piezoelectric materials, Clark et al. (1998):

\[
\begin{align*}
\{D\} &= \{e\} \{S\} + \{e^S\} \{E\} \\
\{\sigma\} &= \{e^E\} \{S\} - \{e\}^T \{E\}
\end{align*}
\]

where the superscript ($\cdot^T$) means transpose, ($\cdot^S$) means that the values are measured at constant strains, ($\cdot^E$) means that the values are measured at constant electric field, $\{\sigma\}$ is the stress tensor, $\{D\}$ is the electric displacement vector, $\{S\}$ the strain tensor, $\{E\}$ is the electric field, $\{e^E\}$ is the elasticity tensor at constant electric field, $\{e\}$ is the dielectric permittivity tensor at constant mechanical strain (permittivity matrix), and $\{e^S\}$ is the dielectric tensor at constant strain.

The electromechanical coupling of piezoelectric material and the host structure can be modeled by different approaches. Finite Element Method (FEM) is an approximation of an integral formulation, by a linear algebraic formulation, where the coefficients are integral evaluations on the sub-area of the resolution area. In this paper, the equations of motion for a Kirchhoff plate model with bonded piezoelectric, represented by Euler-Bernoulli beam elements, were derived using the Rayleigh-Ritz formulation. The assumed displacement field shapes within the elastic body and the electric potential field shapes were combined through the piezoelectric properties to form a set of coupled electromechanical equations of motion.

The choice of the coordinates in finite elements modeling is arbitrary, but in general two coordinate systems are used: nodal and modal coordinates. However, the representation in nodal coordinates is seldom used in control design, once the
estimate of modal parameters is easier to be implemented. The modal state-space is a standard used by structural control engineering, and the linear model is described by the following equations:

\[
\begin{align*}
\dot{x}(t) &= Ax(t) + Bu(t) \\
y(t) &= Cx(t)
\end{align*}
\]  

(2)

where \(A\), \(B\) and \(C\) are dynamic matrix (2x2 order), input matrix, and output matrix, respectively, \(x(t)\) is the state-space vector, \(y(t)\) is the output vector and, \(u(t)\) is the input vector. The modal state-space realization is characterized by the block-diagonal dynamic matrix and the related input and output matrices:

\[
A = \text{diag}(A_{mi}),
\]

\[
B = \begin{bmatrix} B_{m1} & B_{m2} & M & B_{mn} \end{bmatrix}^T,
\]

\[
C = \begin{bmatrix} C_{m1} & C_{m2} & L & C_{mn} \end{bmatrix}
\]  

(3)

where \(i = 1, 2, \ldots, n\), \(A_{mi}\), \(B_{mi}\) and \(C_{mi}\) are 2x2, 2x(input number) and (output number)x2 blocks, respectively. These blocks can be obtained by several different forms and, also, it is possible to represent in another base through a linear transformation. One possible form to block \(A_{mi}\) is:

\[
A_{mi} = \begin{bmatrix} -\zeta_i \omega_i & \omega_i \\ -\omega_i & -\zeta_i \omega_i \end{bmatrix}
\]  

(4)

The state vector \(x(t)\) of the modal coordinates consists of \(n\) independent components, \(x_i\), that represent a state of each mode. The \(x_i\) (ith state) component is as follow:

\[
x_i(t) = \begin{bmatrix} q_{mi} \\ q_{moi} \end{bmatrix}, \quad \text{where} \quad q_{moi} = \zeta_i q_{mi} + \dot{q}_{mi}/\omega_i
\]  

(5)

The use of FEM demands high number of degrees of freedom (dof). So, the order of the representation is generally very large, causing numeric difficulties. Besides, the complexity and controller performance based on a model depend, above all, of the order of the plant in study. A reduced-order model can be obtained truncating the states. Let \(x(t)\) and the triple state \((A, B, C)\) be partitioned considering the canonical modal decomposition. From the Jordan canonical form can be obtained:

\[
\begin{bmatrix} \dot{x}_r(t) \\ \dot{x}_i(t) \end{bmatrix} = \begin{bmatrix} A_r & 0 \\ 0 & A_i \end{bmatrix} \begin{bmatrix} x_r(t) \\ x_i(t) \end{bmatrix} + \begin{bmatrix} B_r \\ B_i \end{bmatrix} u(t)
\]

\[
y(t) = \begin{bmatrix} C_r \\ C_i \end{bmatrix} \begin{bmatrix} x_r(t) \\ x_i(t) \end{bmatrix}
\]  

(6)
where the subscript \( r \) is relative to retained state (modes of reduced model) and \( t \) is relative to truncated states (modes of residual model). The main problem is to find the order and the retained states that best reproduces the response of the complete system. The choice depends mainly on the definition of the reduction index used. In general, system norms are used to evaluate the reduction error. Different methods of model reduction through LMI were proposed for cases of local and global optimization, Assunção et al. (2002). In the present work the \( H_\infty \) norm is used as performance index. The norm of the ith natural mode \((A_{mi}, B_{mi}, C_{mi})\) can be estimated in different forms. In this article is used linear matrix inequalities (LMI) to compute the \( H_\infty \) norm. The \( H_\infty \) norm system can be found from the following optimization problem:

\[
\begin{bmatrix}
A^T P + PA + CC^T & PB \\
B^T P & -\mu
\end{bmatrix} < 0,
\]

subject to \( P > 0 \) and \( \mu > 0 \)

where \( P \) is a symmetric positive defined matrix, and \( \mu \) is a scalar, Gonçalves (2002). This norm also corresponds to the peak gain of frequency response function. The \( H_\infty \) norm system is the largest value of the norm of each mode. The \( H_\infty \) reduction error is defined as:

\[
e = \|G - G_r\|_\infty
\]

where \( G \) and \( G_r \) are transfer functions of the nominal system and of the reduced-order model, respectively.

**PLACEMENT OF SENSOR/ACTUATOR USING \( H_\infty \) NORM**

The optimal placement problem consists of determining the location of a small set of actuators and sensors such that \( H_\infty \) norm system is the closest possible of the norm system using a large set of actuator and sensor. The \( H_\infty \) placement index \( \sigma_{ik} \) evaluates the \( k \)th actuator (sensor) at the \( i \)th mode. It is defined in relation to every modes and every admissible actuators, as:

\[
\sigma_{ik} = \frac{\|G_k\|_\infty}{\|G_i\|_\infty}, \quad k = 1, \ldots, s, \quad i = 1, \ldots, n
\]

where \( s \) is the number of actuators and \( n \) is the number of considered modes. It is convenient to represent the placement index as a placement matrix:
where the $kth$ column consists of indexes of the $kth$ actuator for each mode, and the $ith$ row is a set of the indexes of the $ith$ mode for each actuator. The actuators with small indexes can be removed as the least significant. The largest value indexes are optimal placement actuator. Similarly, it is possible to determine the optimal placement of the sensors.

**NUMERICAL APPLICATION**

To verify the proposed methodology, a computer program, called SMARTSYS, was implemented using MATLAB® code. This program was designed to find the optimal positions of PZT actuators for vibration attenuation of the two first modes in a clamped-free-free-free aluminum plate. The plate has 0.2x0.2x0.003 m of length, width, and thickness, respectively, and it was discretized in 10x10 square plate elements with four nodes and three degree of freedom per node, fig. 1. The properties of aluminum plate are: Young’s modulus = 70 Gpa; mass density = 2710 kg.m$^{-3}$; and Poisson’s coefficient = 0.3.

To test the proposed optimization methodology, the first step was to find the optimal placement of two pairs of PZT, bonded on both sides of the plate surface. Figure 1 shows the plate with PZT actuators bonded in the optimal positions, case (b). The plate without PZT was designed by case (a).

The placement index of each candidate actuator for the first and second modes are shown in fig. 2. The largest value index for the actuators correspond to positions where the actuators are more effective.

![Figure 1. Plate structure with bonded PZTs](image1)

![Figure 2. Placement index versus actuator location](image2)
The numbers 1 to 121, fig. 1, represent the nodes sequency of the model, and it was considered that each PZT can be positioned among these nodes. The number of electrical dof changes as a function of the number of piezoelectric patch elements considered (2 dof by PZT), Lopes et al. (2003). The properties of PZT are: Young’s modulus = 60 Gpa; mass density = 7650 kg.m⁻³; dielectric constant = 190e-12 m.V⁻¹; dielectric permitivity = 30.705 C/m²; elasticity = 1.076e11 m/N²; and permitivity of space = 7.33e-9 F/m.

To evaluate the implemented model, the results were compared with the ANSYS® model discretized in solid elements. The FRF of the system modeled at SMARTSYS (excitation and measurement at node 91) without PZT, case (a), and considering the electromechanical coupling between the PZT and the host structure, cases (b), are shown in fig. 3. Figure 4, shows the frequency response obtained by ANSYS® program for cases (a) and (b).

Figures 3 and 4 show the influence of electromechanical coupling in the dynamic response of the structure. The differences between the results of the programs can be explained, mainly, because the modeled PZT in the ANSYS program has a short dimension relationship, and the solid element are not appropriate to represent the thin structures behavior.

**EXPERIMENTAL APPLICATION**

**Figure 5. Schematic setup of the experimental test**

**Figure 6. Test structure**
In this section, the finite element models and the optimal placement of actuators are experimentally tested. The test structure consists of an aluminum cantilever plate with two pairs of bonded PZT actuators, as shown in figures 5 and 6. The dimensions of the plate are: 0.2x0.2x0.002 m of length, width, and thickness, respectively. The aluminum and PZT properties were described in the previous section. The experimental results were verified by using two different excitations: (1) by hammer impact and (2) by PZTs actuators.

Figures 7 to 9 show the FRF of the experimental tests for both types of excitation. The results were compared with the numerical model dicretized as shown in fig. 5. The PZT actuators, modelled by beam elements, were positioned between nodes 13 - 14 and 101 - 102 of the plate.

The hammer impact test showed a great concordance between the experimental and simulated FRF. When the excitation was done by PZT we can observe some variation in high frequency range. The force transferred from the PZT to the plate considers only two opposing moments in the ends of the beam element. It is not the real way that the excitation is transferred to the structure so, a further approach must consider this information. However, as shown in the fig. 10, the modeling considering the influence of the electromechanical coupling reduced the differences between

![Figure 7. FRF using hammer impact test (input: node24 - output: node 21)](image1)

![Figure 8. FRF using PZT actuators for excitation (output: node 121)](image2)

![Figure 9. FRF using PZT actuator for excitation (output: node 99)](image3)

![Figure 10. Difference between natural frequencies](image4)
experimental and numerical model for the SMARTSYS program, while the ANSYS® program presented a worse performance, especially for modes of higher order.

CONCLUSIONS

This paper presents a methodology using $H_\infty$ norm, computed by LMI, for optimal placement of sensor/actuator in a plate structure. To evaluate the model, the results were compared with the ANSYS® program and experimental tests. In both programs, the PZT elements were positioned at the optimal positions considering the two first modes of the structure. It was verified the bigger influence of the electromechanical coupling at the two first modes for actuators in optimal positions. The experimental results showed a great concordance with simulated results, although there are limitations in the beam elements, used to model the PZT actuator, regarding to application of moments.

REFERENCES


