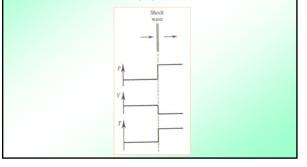
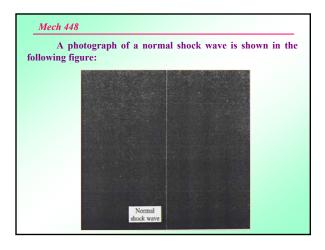


**SHOCK WAVES:** It has been found experimentally that, under some circumstances, it is possible for an almost spontaneous change to occur in a flow, the velocity decreasing and the pressure increasing through this region of sharp change. The possibility that such a change can occur actually follows from the analysis given below. It has been found experimentally, and it also follows from the analysis given below, that such regions of sharp change can only occur if the initial flow is supersonic. The extremely thin region in which the transition from the supersonic velocity, relatively low pressure state to the state that involves a relatively low velocity and high pressure is termed a shock wave.



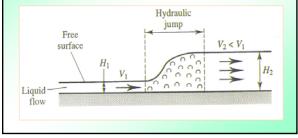
The changes that occur through a normal shock wave, i.e., a shock wave which is straight with the flow at right angles to the wave, is shown in the following figure:

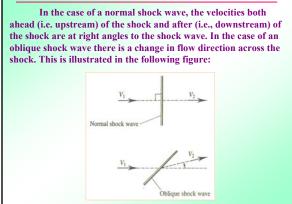


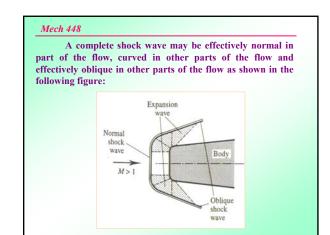


## **Mech 448**

A shock wave is extremely thin, the shock wave normally only being a few mean free paths thick. A shock-wave is analogous in many ways to a "hydraulic-jump" that occurs in free-surface liquid flows, a hydraulic jump being shown schematically below. A hydraulic jump occurs, for example, in the flow downstream of a weir.







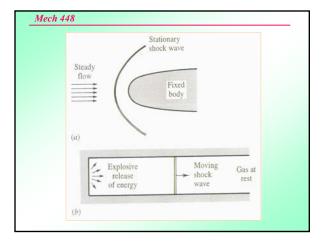
### **Mech 448**

Because of their own importance and because, as will be shown later, the oblique shock relations can be deduced from those for a normal shock wave, the normal shock wave will be first be considered in the present chapter. Oblique shock waves will then be discussed in the next chapter. Curved shock waves are relatively difficult to analyze and they will not be discussed in detail in the present course.

Normal shock waves occur, for example, in the intakes to the engines in some supersonic aircraft, in the exhaust system of reciprocating engines, in long distance gas pipe-lines and in mine shafts as a result of the use of explosives.

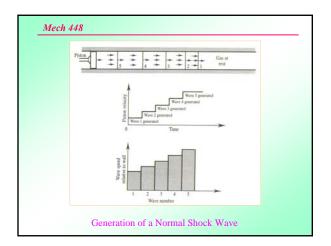
### **Mech 448**

When a normal shock wave occurs, for example, in a steady flow through duct, it can be stationary with respect to the coordinate system which is fixed relative to the walls of the duct. Such a shock wave is called a stationary shock wave since it is not moving relative to the coordinate system used. On the other hand, when a sudden disturbance occurs in a flow, such as, for example, the sudden closing of a valve in a pipe-line or an explosive release of energy at a point in a duct, a normal shock wave can be generated which is moving relative to the duct walls. This is illustrated in the following figure.



#### Mech 448

To illustrate how a shock wave can form, consider the generation of a sound wave as discussed earlier. It was assumed that there was a long duct containing a gas at rest with a piston at one end of this duct that was initially at rest. Then, at time 0, the piston was given a small velocity into the duct giving rise to a weak pressure pulse, i.e., a sound wave, that propagated down the duct (see following figure).



If dV is the velocity given to the piston, which is, of course, the same as the velocity of the gas behind the wave, then the increase in pressure and temperature behind the wave are equal to  $\rho a \, dV$  and  $[(\gamma - 1) T \, dV / a]$  respectively. Since  $\rho$ , a, and T are all positive, this shows that the pressure and temperature both increase across the wave. It was also shown that the velocity at which the wave moves down the duct is equal to  $\sqrt{\gamma RT}$ , which is by definition the speed of sound. Therefore, since the temperature increases across the wave, the speed of sound behind the wave will be a + da, where da is positive. Now consider what happens if some time after the piston is given velocity dV into the duct, its velocity is suddenly again increased to 2 dV. As a result of the second increase in piston speed, a second weak pressure wave will be generated that follows the first wave down the duct as shown in the above figure.

## **Mech 448**

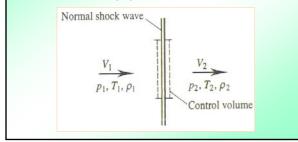
This second wave will be moving relative to the gas ahead of it at the speed of sound in the gas through which it is propagating. But the gas ahead of the second wave has velocity dV. Hence, the second wave moves relative to the duct at a velocity of a + da + dV. But, the first wave is moving at a velocity of a relative to the duct. Therefore, since both da and dV are positive, the second wave is moving faster than the first wave and, if the duct is long enough, the second wave will overtake the first wave. But the second wave cannot pass through the first wave. Instead, the two waves merge into a single stronger wave. If, therefore, the piston is given a whole series of step increases in velocity, a series of weak pressure waves will be generated which will all eventually overtake each other and merge into a single strong wave if the duct is long enough, i.e., a moving normal shock wave will be generated.



The analysis of stationary normal shock waves will first be considered and then the application of this analysis to moving normal shock waves will then be discussed.

# Mech 448

**STATIONARY NORMAL SHOCK WAVES:** Attention will first be given to the changes that occur though a stationary normal shock wave. In order to analyze the flow though a stationary normal shock wave, consider a control volume of the form indicated in the following figure:



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i.e.:

This control volume has a cross sectional area of A normal to the flow direction. The shock wave relations are obtained by applying the laws of conservation of mass, momentum, and energy to this control volume. Conservation of mass gives:

$$m = \rho_1 V_1 A = \rho_2 V_2 A$$

 $\rho_1 V_1 = \rho_2 V_2$ 

or

Next consider conservation of momentum. Since the only forces acting on the control volume in the flow direction are the pressure forces, conservation of momentum applied to the control volume gives:

$$p_1 - p_2 = \rho_1 V_1 (V_2 - V_1)$$
$$p_1 - p_2 = \rho_2 V_2 (V_2 - V_1)$$

$$\frac{Mech \ 448}{}$$
These two equations can be rearranged to give:  

$$V_1 V_2 - V_1^2 = \frac{p_1 - p_2}{\rho_1}$$
and:  

$$V_2^2 - V_2 V_1 = \frac{p_1 - p_2}{\rho_2}$$

$$\tilde{m}$$
Adding these two equations together then gives:  

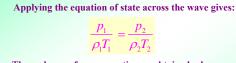
$$V_2^2 - V_1^2 = \left(p_1 - p_2\right) \left(\frac{1}{\rho_1} + \frac{1}{\rho_2}\right)$$

### **Mech 448**

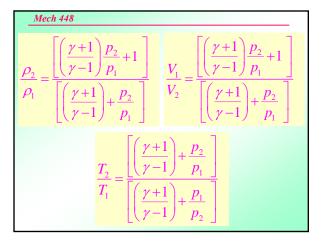
Lastly, consider the application of conservation of energy to the flow across the shock wave. Because one-dimensional flow is being considered there are no changes in the flow properties in any direction that is normal to that of the flow and, because the upstream and downstream faces of the control volume lie upstream and downstream of the shock wave, there are no temperature gradients normal to any face of the control volume. The flow through the control volume is, therefore, adiabatic and the energy equation, therefore, gives:

$$\frac{V_1^2}{2} + c_p T_1 = \frac{V_2^2}{2} + c_p T_2 = c_p T_0 = \text{constant}$$

The stagnation temperature therefore does not change across the shock.



The above four equations obtained by applying conservation of mass, conservation of momentum, conservation of energy and the equation of state can be combined to give the following:



## Mech 448

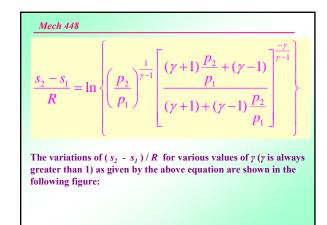
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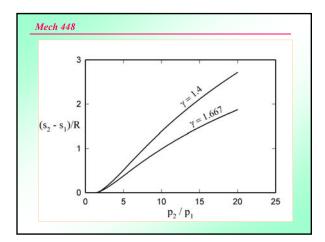
The above three equations give the density, velocity and temperature ratios,  $\rho_2 / \rho_1$ ,  $V_2 / V_1$ , and  $T_2 / T_1$ , across a normal shock wave in terms of the pressure ratio,  $p_2 / p_1$ , across the shock wave. The pressure ratio,  $P_2 / P_1$ , is often termed the strength of the shock wave. This set of equations is often termed the Rankine-Hugoniot normal shock wave relations.

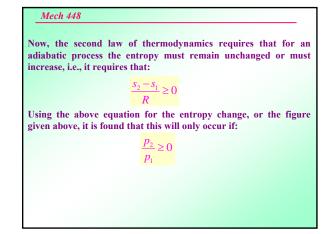
While the application of conservation of mass, momentum and energy principles shows that a shock wave can exist, it does not indicate whether the shock can be either compressive (i.e.,  $p_2/p_1 > 1$ ) or expansive (i.e.,  $p_2/p_1 < 1$ ). To examine this, the second law of thermodynamics must be used. Now the entropy change across the shock wave is given by:

$$s_2 - s_1 = c_p \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{p_2}{p_1}\right)$$

Using the relations for  $T_2/T_1$  and  $p_2/p_1$  given above then gives:







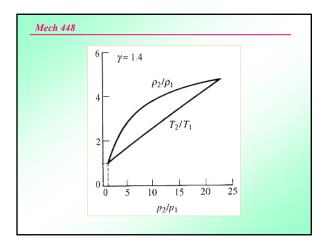
# Mech 448

It therefore follows that the shock wave must always be compressive, i.e., that  $p_2 / p_1$  must be greater than 1, i.e., the pressure must always increase across the shock wave. Using the equations for the changes across a normal shock then shows that that the density always increases, the velocity always decreases and the temperature always increases across a shock wave.

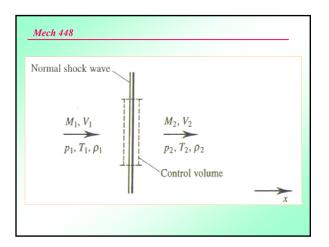
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The entropy increase across the shock is, basically, the result of the fact that, because the shock wave is very thin, the gradients of velocity and temperature in the shock are very high. As a result, the effects of viscosity and heat conduction are important within the shock leading to the entropy increase across the shock wave.

Because the flow across a shock is adiabatic, the stagnation temperature does not change across a shock wave. However, because of the entropy increase across a shock, the stagnation pressure always decreases across a shock wave.



NORMAL SHOCK WAVE RELATIONS IN TERMS OF MACH NUMBER: While the relations derived in the previous section for the changes across a normal shock in terms of the pressure ratio across the shock, i.e., in terms of the shock strength, are the most useful form of the normal shock wave relations for some purposes, it is often more convenient to have these relations in terms of the upstream Mach number  $M_1$ . To obtain these forms of the normal shock wave relations, it is convenient to start again with a control volume across the shock wave such as that shown in the following figure and to again apply conservation of mass, momentum and energy to this control volume but in this case to rearrange the resulting relations in terms of Mach number.





In writing the conservation laws, no generality is lost by taking the area of the control volume parallel to the wave as unity. Conservation of mass then gives:

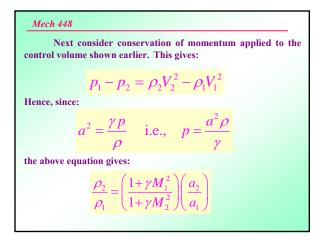
$$\rho_1 V_1 = \rho_2 V$$

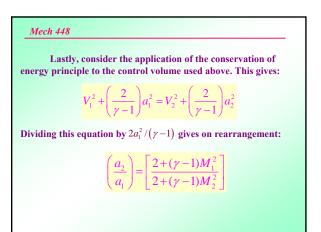
Dividing this equation by  $a_1$  then gives:

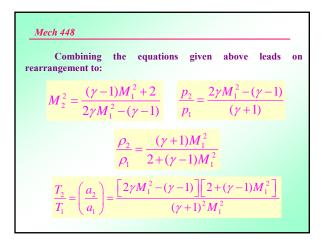
$$p_1 \frac{V_1}{a} = \rho_2 \frac{V_2}{a} \frac{a_2}{a}$$

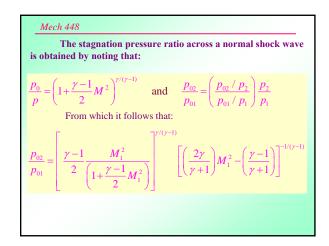
which can be rewritten in terms of Mach numbers as:

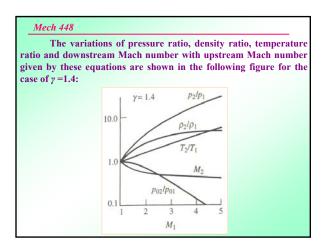
 $\frac{\rho_2}{\rho_1} = \frac{M_1}{M_2} \frac{a_1}{a_2}$ 

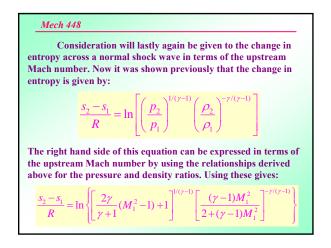


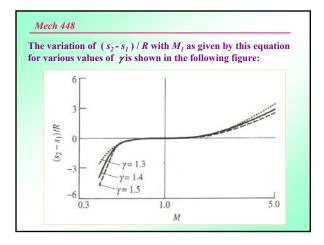












Now as discussed before , the second law of thermodynamics requires that for an adiabatic process the entropy must remain unchanged or must increase, i.e., it requires that:

$$\frac{s_2 - s_1}{R} \ge 0$$

It will be seen from the results given in the above figure that for this to be satisfied it is necessary that:

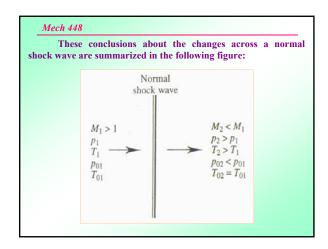
 $M_1 \ge 1$ 

It, therefore, follows that the Mach number ahead of a shock wave must always be greater than 1 and that the shock wave must, therefore, as discussed before, always be compressive, i.e., the pressure must always increase across the shock wave.

Mech 448  
It then follows from:  

$$M_2^2 = \frac{(\gamma - 1)M_1^2 + 2}{2\gamma M_1^2 - (\gamma - 1)}$$
that:  

$$M_2 \le 1$$
i.e., the flow downstream of a normal shock wave will always be subsonic.

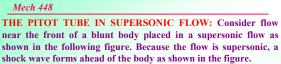


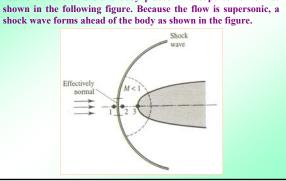
**NORMAL SHOCK WAVE TABLES:** A number of sets of tables and graphs are available which list the ratios of the various flow variables such as pressure, temperature and density across a normal shock wave and the downstream Mach number as a function of the upstream Mach number for various gases, i.e. for various values of  $\gamma$ . An example of a set of tables is shown below.

The values in these tables and graphs are, of course, derived using the equations given in the previous section. As with isentropic flow, instead of using tables, it is often more convenient to use a computer program to find the changes across a shock wave. Alternatively, most calculators can be easily programmed to give results for a normal shock wave.

_	rmal S		Table	s for γ	y = 1.4	ļ	M <sub>4</sub> 1.62 1.64 1.64 1.64 1.64 1.70 1.72 1.74 1.78 1.80 1.80 1.90 1.94	Mg 8.642 57 8.6556 77 8.6511 19 0.6455 79 8.6405 54 8.6505 51 0.625 70 0.625 70 0.625 70 0.625 70 0.621 04 0.635 50 0.642 104 0.605 63 0.690 17 0.595 62 0.595 62 0.595 62 0.595 62	Ps/Ps 2.895 13 2.971 20 3.048 20 3.2951 00 3.295 00 3.295 00 3.295 40 3.365 53 3.467 20 3.529 40 3.529 40 5.529 40 5.529 40 5.529 40 5.529 400 400 500 50000000000000000000000000	T <sub>b</sub> (T <sub>6</sub> ) 1.401 82 1.415 78 1.429 85 1.448 53 1.422 54 1.421 53 1.422 54 1.516 92 1.516 92 1.516 99 1.546 99 1.547 99 1.517 90 1.577 90 1.577 91 1.425 57	2045.25 2.085.25 2.098.43 2.131.83 2.164.86 2.197.73 2.200.40 2.202.97 2.202.97 2.202.97 2.202.97 2.202.97 2.202.97 2.209.93 2.422.44 2.453.73 2.444.81 2.213.548 2.2464.51	Phil/Phil 0.38745 0.37942 0.36544 0.36572 0.34736 0.33886 0.33024 0.32151 0.31248 0.30745 0.33540 0.33540 0.37455 0.37455 0.37655 0.37655	Pub 3.886 58 3.590 28 4.855 85 4.855 85 4.137 90 4.255 13 4.319 10 4.488 14 4.488 14 4.488 14 4.600 31 4.781 34 4.855 24 4.855 24 4.955 24 5.967 25 5.967 25 5
Nor	rmal S	Shock	Table	s for γ	y = 1.4		1.64 1.68 1.70 1.72 1.74 1.75 1.80 1.80 1.80 1.84 1.88 1.90 1.94	8.636 17 8.631 19 0.645 79 0.646 54 0.635 45 0.625 76 0.625 76 0.625 76 0.625 76 0.612 69 0.612 69 0.612 69 0.617 10 0.995 52 0.995 52 0.995 52	2.97129 3.04829 3.12613 3.221409 3.224409 3.58553 3.44729 3.62540 3.61533 3.64729 3.62740 3.62740 3.62740 3.62940 3.64740 3.50740 3.50940 4.64540 4.13413	1.413 78 1.42943 1.444 03 1.428 33 1.472 34 1.487 27 1.501 92 1.536 69 1.546 59 1.546 59 1.546 79 1.547 70 1.597 90 1.592 57	2,098.43 2,131.63 2,164.86 2,197.72 2,200.60 2,202.89 2,295.20 2,295.	0.879-92 0.872-01 0.863-72 0.847-76 0.847-76 0.878-86 0.870-24 0.821-51 0.812-88 0.800-76 0.994-76 0.795-80 0.776-55 0.547-36	3300 28 4.855 85 4.337 90 4.225 83 4.310 80 4.488 94 4.578 34 4.578 34 4.585 34 4.585 34
Nor	rmal S	Shock	Table	s for γ	y = 1.4		1.66 1.68 1.70 1.72 1.76 1.80 1.80 1.80 1.80 1.80 1.80 1.80 1.90 1.94	8.631 19 8.645 79 0.640 54 0.635 545 0.635 545 0.625 70 0.621 64 0.635 50 0.642 164 0.636 50 0.640 63 0.990 57 0.395 62 0.396 77	3.048 20 3.126 13 3.204 40 3.204 40 3.506 53 3.447 20 3.623 40 3.623 40 3.623 40 3.627 40 3.647 20 3.627 40 3.627 40 3.527 40 3.529 40 4.346 50 4.346 50	1.429.83 1.444.03 1.458.53 1.472.25 1.501.92 1.516.69 1.531.98 1.546.59 1.547.70 1.577.60 1.577.90 1.577.91 1.425.57	2.131 83 2.164 86 2.197 72 2.206 60 2.362 89 2.362 89 2.357 31 2.379 32 2.490 93 2.425 73 2.445 83 2.445 83 2.445 15 68 2.546 32	0.87240 0.86354 0.85572 0.84736 0.83886 0.83024 0.82151 0.81248 0.80076 0.85540 0.87545 0.75655 0.75655	4.853 85 4.137 90 4.223 83 4.310 83 4.310 80 4.488 94 4.578 24 4.578 24 4.569 51 4.751 54 4.555 24 4.5980 60
NOT	mai s	Snock	Table	s Ior γ	y = 1.4		1.68 1.75 1.75 1.78 1.80 1.80 1.80 1.84 1.86 1.90 1.94	0.545 79 0.540 54 0.525 45 0.525 70 0.525 70 0.521 04 0.516 90 0.512 04 0.507 80 0.507 80 0.507 80 0.507 85 0.599 57 0.395 52 0.590 77	3.126 13 3.207 00 3.208 40 3.508 53 3.447 20 3.628 53 3.447 20 3.629 40 3.627 40 3.627 40 3.703 20 3.505 40 4.045 30 4.134 13	1.444.07 1.429.33 1.472.34 1.487.27 1.501.92 1.531.98 1.546.99 1.546.99 1.544.73 1.547.90 1.542.77 1.597.90	2.164.16 2.197.72 2.200.60 2.362.89 2.395.20 2.395.20 2.399.13 2.390.83 2.422.44 2.453.73 2.484.81 2.515.68 2.546.32	0.363 94 0.835 72 0.347 36 0.830 24 0.821 51 0.812 68 0.800 76 0.796 35 0.715 69 0.776 35	4.137 90 4.225 83 4.310 83 4.398 90 4.488 04 4.578 24 4.660 31 4.551 24 4.855 24 4.940 60
							1,79 1,72 1,78 1,78 1,80 1,80 1,80 1,80 1,80 1,90 1,90 1,94	0.640 54 0.635 45 0.630 51 0.625 70 0.625 70 0.616 50 0.612 59 0.607 80 0.607 80 0.909 57 0.395 62 0.395 62	3.201 00 3.204 80 3.305 53 3.447 20 3.529 80 3.613 53 3.697 80 3.703 20 3.703 20 3.703 20 4.046 80 4.046 13	1,472 74 1,487 27 1,501 92 1,516 89 1,516 99 1,546 99 1,547 90 1,592 39 1,597 90 1,592 91 1,625 57	2 236 40 2 242 89 2 245 20 2 317 31 2 339 22 2 390 93 2 452 44 2 453 73 2 484 81 2 546 32	0.347 36 0.838 36 0.830 24 0.821 51 0.812 68 0.900 36 0.796 35 0.796 35 0.357 36	4.310 (3) 4.398 90 4.488 04 4.579 24 4.600 31 4.751 84 4.855 24 4.940 00
							1.74 1.78 1.80 1.82 1.84 1.88 1.99 1.97 1.94	0.630 51 0.625 70 0.625 70 0.616 50 0.607 80 0.607 80 0.507 80 0.509 57 0.595 62 0.395 52	3.365 53 3.447 20 3.529 80 3.613 33 3.697 80 3.703 20 3.809 53 3.936 80 4.045 80 4.134 13	1.487 27 1.501 92 1.516 69 1.516 59 1.546 59 1.546 59 1.547 73 1.517 60 1.592 39 1.667 91 1.625 57	2 342 89 2 245 20 2 327 31 2 339 22 2 399 93 2 422 44 2 453 73 2 484 81 2 515 68 2 546 32	0.838 86 0.830 24 0.821 51 0.812 68 0.807 76 0.794 76 0.785 69 0.776 35 0.367 36	4.398 90 4.488 04 4.578 24 4.600 31 4.761 84 4.855 24 4.940 60
							1.76 1.78 1.80 1.82 1.84 1.86 1.90 1.90 1.92	0.62570 0.62104 0.63650 0.60780 0.60780 0.50957 0.59562 0.39077	3.447 20 3.529 80 3.613 33 3.697 80 3.703 20 3.809 53 3.936 80 4.045 90 4.134 13	1.501 92 1.516 69 1.511 58 1.546 59 1.361 73 1.517 60 1.512 59 1.607 91 1.625 57	2 295 20 2 327 31 2 399 32 2 402 44 2 453 73 2 484 81 2 515 68 2 546 32	0.830 24 0.821 51 0.812 68 0.900 76 0.794 76 0.785 69 0.776 35 0.367 36	4,488 14 4,579 24 4,660 31 4,751 54 4,855 24 4,949 69
							1.78 1.80 1.82 1.84 1.86 1.86 1.90 1.92 1.94	0.821 04 0.616 50 0.812 09 0.607 80 0.607 80 0.607 80 0.999 37 0.595 62 0.595 62	1.539.80 1.613.53 3.697.80 3.703.20 3.809.53 3.956.80 4.045.90 4.134.13	1,516.69 1,516.59 1,546.59 1,546.73 1,517.60 1,512.39 1,607.91 1,625.57	2.327 31 2.399 22 2.390 93 2.422 44 2.453 73 2.484 81 2.515 68 2.546 32	0.821 51 0.812 68 0.900 76 0.794 76 0.785 69 0.776 35 0.307 36	4.579 24 4.660 31 4.761 54 4.855 24 4.949 69
							1.80 1.82 1.84 1.86 1.85 1.90 1.92 1.94	0.616 50 0.612 09 0.607 80 0.601 63 0.599 37 0.595 62 0.595 62	3.613 33 3.697 80 3.793 20 3.899 53 3.956 80 4.045 80 4.134 13	1.531 58 1.546 59 1.541 73 1.517 60 1.592 39 1.607 91 1.625 57	2,319,22 2,390,93 2,422,44 2,453,73 2,484,81 2,515,68 2,546,32	0.812.68 0.905.76 0.794.76 0.785.69 0.776.35 0.767.36	4,660 31 4,751 54 4,855 34 4,949 69
							1.82 1.84 1.86 1.85 1.90 1.97 1.94	0.812.09 0.607.80 0.601.63 0.399.37 0.395.62 0.391.77	3.49780 3.78320 3.89953 3.95680 4.04580 4.13413	1.546 59 1.541 73 1.517 00 1.592 39 1.607 91 1.625 57	2,390 93 2,422 44 2,453 73 2,484 81 2,515 68 2,546 32	0.807 76 0.794 76 0.785 69 0.776 55 0.767 36	4,761 84 4,855 34 4,949 69
							1.84 1.85 1.90 1.92 1.94	0.60780 0.60563 0.39937 0.39562 0.39177	3.763 20 3.869 53 3.956 80 4.045 00 4.134 13	1.561 73 1.577 60 1.592 39 1.607 91 1.625 57	2,422 44 2,453 73 2,484 81 2,515 68 2,546 32	0.794 76 0.785 69 0.776 55 0.367 36	4,95534
							1.86 1.85 1.90 1.97 1.94	0.603 63 0.399 57 0.395 62 0.391 77	3.869.53 3.956.80 4.045.00 4.134.13	1.577-00 1.592 39 1.607 91 1.625 57	2,453 73 2,484 81 2,515 68 2,546 32	0.785 89 0.776 55 0.767 36	4,543 cp
							1.88 1.90 1.92 1.94	0.395 62 0.391 77	4.045/00 4.134/13	1.607 91 1.625 57	2.515.68 2.546.32	0.36736	
							1.92	0.591 77	4.13413	1.625.57	2.546 32		
							1.94		4.13413	1.625.57	2.546.32		5.14177
													5.239.40 5.338.06
									4.234.20 4.315.20	1.639.35	2,576 75 2,606 95	0.745 84 0.719 54	5.308.08
							1.96	0.584 37 0.580 82	4.31520	1.675.32	2,606 97 2,636 92	0.739.21	5.538.60
							2.00	0.577.35	4,300.00	1.687.50	2.666 67	0.728.87	5.640-64
_						_	2.82	0.573 97	4,595.80	1.703 82	2,606 18	0.711.53	5.743.32
ς	- M <sub>1</sub>	nin	$T_3/T_1$	nin	PealPea	nuin	2.04	0.170.68	4.688.53	1.729.27	2,725.46	0.79218	5.847 26
10	1.005-00	1.000.00	1.000-00	1.000.00	1.000.00	1.892.95	2.06	0.567.47	4,784.19	1.756.86	2,754.51	0.002.84	5.952.25
12	0.990.52	1.04713	1.013.25	1.033-44	0.999 99	1.907.90	2.08	0.56435	4,890 90	1.753.99	2,783.32	0.683.51	6.058.29
14	0.962.05	1.095.20	1.026.34	1.067.09	0.999 92	1.984.42	2.10	0.56128	4.978.33 5.076.79	1.778-45	2,811 90 2,840 24	0.674.20 0.664.92	6.215.50
16	0.94445	1.144.20	1.039.30	1.100.92	0.999 75	2.002.45	2.12	0.558.29 0.555.38	5.176.19	1,804,79	2,848,34	8.635.67	6.382.68
16	0.92771 0.91177	1.194 13	1.05217	1.13492	0.999 43	2.081 94	2.14	0.555.38	5,176.29	1.804.79	2,898.24	0.646.41	6.412.90
10	0.911 77 0.896 56	1.245 00	1.064.94	1,149-08	0.996 93 0.996 21	2.132 85 2.185 13	2.38	0.549 77	5,377.79	1,839,30	2.923.83	0.637.27	6.80416
i4	0.882.04	1.349.53	1.090 27	1.237 79	0.998 21 0.997 26	2.185 13 2.218 77	2.39	0.547.06	1.479.99	1.856.86	2.951 22	0.62614	6.716-47
6	0.868.16	1.403.20	1.162.87	1.272.31	0.996-0.5	2.299.72	2.22	0.544.40	5.583 13	1.874.56	2,978.36	0.619-05	6.829.83
8	0.85488	1.457 80	1.115-64	1.306.93	0.994 17	2,340.98	2.34	0.541 82	5.68719	1.892.40	3.085 27	0.630.92	6.944 23
20	0.842 17	1.513 33	1.127.99	1.34161	0.99280	2,407,50	2.36	0.539.30	5.792.19	1,930-40	3.031.93	0.601-05	7,859-67
12	0.829.99	1.569 80	1.140.54	1.576.36	0.990 73	2.466 28	2.28	0.536.83	5,898 13	1.928.53	3.038.36 3.084.55	0.59214 0.58330	7,17613
14	0.818 30	1.627 20	1.153.09	1.41116	0.988.26	2.526 29	2.30 2.32	0.532-05	6.112.79	1.946.90	3.084 55 3.110 49	0.583 30 0.574 52	7,299.87
26	0.807.09	1.685 53	1.165.66	1.445.99	0.985.68	2.587 53	2.32	0.532-05	6.221.53	1,963.78	3.136.20	0.565 81	7,531.84
28 10	0.796 31 0.785 96	1.344 80	1.179.25	1.480.84	6.982.68	2.649.96	2.56	0.527.49	6.331.19	2,002,48	3.141.67	0.55718	7.652.49
2	0.775.00	1,865 13	1,203.55	1.513-69	6.979.37	2.713 59 2.778 40	2.38	0.525 28	6.44179	2.421.33	3.186 90	0.548 62	7,77418
ii ii	0.76541	1.828.20	1.216.26	1.530.55	6.975 75	2,718.40	2.40	0.52512	6.553.53	2.840.33	3,211,89	0.54014	7,896 91
16. L	0.75718	1.991 20	1.229.00	1.620.18	6.967.58	2.911.52	2.42	0.521.00	6.66579	2.019-47	3.236.65	0.531 75	8.029.67
	0.748.29	2,85513	1.240.81	1.654.94	6.963.04	2,979.80	2.64	0.518 94	6.77919	2,878 76	3.261 17	0.525 44	8.14348
0	0.739.71	2.120.00	1.254.69	1.689-65	0.958 19	3.049.23	2.46	0.516 91 0.514 95	6.893 53 7.008 79	2.098.19 2.117.77	3.285 46 3.309 51	0.515 21 0.507 87	8.398.21
0	0.731 44	2.1185 80	1.267.64	1.724.30	0.953.06	3.129.80	2.48	0.514.93 0.512.99	7.006.79 7.134.99	2,117.77	3.309.51	0.307.07	1.52613
14	0.725 45	2.252 53	1.280.66	1,758.88	0.947.65	3.191.49	2.50	0.512 99	7.34212	2.157.37	3,356,92	0.491.05	8.455.09
6	0.71574	2.329 20	1.293 76	1.793.37	0.94196	3.264.30	2.54	0.509 25	7,360 19	2.177.39	3.380.28	0.483 18	8,793.08
8	0.708.29	2.388.80 2.458.33	1.306.95	1.82777	0.926-00 0.929-79	3.538.23 3.413.27	2.56	0.50741	7.479.19	2.197.56	3,403 41	0.475.40	8.91612
2	0.501 09 0.694 13	2.458 33 2.528 80	1.329.22	1.862.07	0.92979	3.413.27	2.58	0.50542	7,599 12	2.217 88	3.426 31	0.467 72	9.048 29
ũ.	0.687 39	2,600 20	1.347.65	1.950.33	0.905.52	3.566.66	2.60	0.503 87	7.719 99	2.258.34	3,448.98	0.460 12	9.181.30
6	0.680 87	2.672.53	1.360 57	1.950.33	0.908-02	3.566501	2.42	0.50216	7.841 79	2.258.95	3.471 43	0.452.63	9,315.44
8	0.674 55 0.668 44	2.74580 2.82090	1.374 22 1.387 97	1.998.08	0.902 55 0.895 20	3.724.44	2.64	0.500.48 0.406.83	7.964 52 8.068 19	2.279 71 2.300 42	3,493.65	0.443 22 0.407 92	9.45043

M.	м,	hih	$T_0/T_1$	nin	PeolPeo	nan	
2.68	0.497.22	8.212.79	2.321.68	3.537.43	0.430 71	9.724 10	
2.59	0.495.63	8.21279	2.542.89	3.558.99	0.423.59	9,862.39	
171	0.494.05	8,464 79	2,364.25	3,580.33	0.406.57	10.001 71	
2.54	0.492 56	8.59219	2.38575	3,601 46	0.409.55	10.142-08	
1.76	0.491.07	8.720.52	2,407.41	3.622.37	0.402.83	10.283 47	
2.78	0.489.60	8.84979	2.429.22	3.643.06	0.39610	10.425.91	
2.80	0.488 17	8.979.99	2.451.17	3.663.55	0.389.46	10.569 37	
2.82	0.486 76	9,11112	2.473.28	3.683.93	0.382.93	10.713 88	
2.84	0.485 38	9.24319 9.37619	2.495.53 2.517.94	3.705.89	0.376.49 0.370.14	10.829 41 11.005 99	
2.86	0.484.02 0.482.69	9.51012	2.540.50	3.743-41	0.363.89	11.153.99	
2.88	0.482.09	9.50012	2.540.50	3.742.86	0.347 73	11.302.23	
2.82	0.481 38	8.780 79	2.586-06	3.78211	0.351 67	11.451.91	
2.94	0.475 84	9:917.52	2.609-07	3.804.17	0.34570	11.602.62	
2.86	0.477.60	18.01519	2.632.23	3.820-92	0.339.82	11.754.36	
2.98	0.476.38	10.193 79	2.655.55	3.838.68	0.334.04	11.907.14	
3.00	0.475 29	18.333.32	2.679.81	3.857.14	0.328.34	12.060.95	
3.05	0.472.30	10.586 24	2.738 33 2.796 60	3.94246	0.314 50 0.301 21	12.450.00	
3.10 3.15	0.469 53 0.466 89	11.044.99	2.598.60	3.946.61	0.201 21 0.288 46	12,845 51 13,247 48	
1.15	0.46435	11,279.98	2.821.99	4.031.49	0.276.23	13.655.90	
3.25	0.461 92	12,156 23	2.98511	4.072.29	0.264 51	14.070 78	
3.30	0.459.59	12.538 32	3.04919	4.112/82	0.253.28	14.492.12	
3.35	0.45735	12,926 23	3.114 22	4.150.71	0.242.52	14,919.91	
3.40	0.455 20	13.319.98	3.190 20	4.188.40	0.232.23	15.35415	
3.45	0.453 14	13.719.96	3.247.15	4.22511	0.22237	15.794.84	
3.50	0.45115 0.44925	14.124.98	3.315.05 3.383.91	4.260 87 4.295 70	0.212.95 0.203.95	16.241 98 16.695 57	
3.35	0.447 41	14.913 31	3.413 72	4.329.62	0.19531	17.155 61	
3.45	0.445.65	15.376.23	3.524.50	4.362.67	6.187.07	17.622 10	
3.70	0.443 95	15,804 98	3.596 24	4.394.86	0.179 19	18.095.04	
3.75	0.44231	16.239.56	3.668 94	4.426.23	0.171 67	18.574.43	
3.80	0.440 73	16.679.98	3.542.60	4.456.79	0.164.47	19.06026	
3.85	0.439.21	17.126 22	3.817.22	4.485.57	0.157.60	19.552.54	
3.90	0.437 74	17.578.34	3.892.81	4.515.58	0.151.00	20.05126	
3.95	0.436.33 0.434.96	18.856 22 18.499 97	3.969.36 4.046.87	4.343 86 4.371 43	0.14475 0.13876	29.556.44 21.068.05	
4.00	0.433.64	18,969 57	4.125.35	4,596.29	0.113.05	21.586.12	
4.10	0.432.36	19.444 99	4.204 79	4.624.48	0.127.56	22.110.64	
4.15	0.47113	19.926.26	4.265.29	4.659.02	0.122.33	22.641.61	
4.20	0.429.94	20.413 35	4.366.57	4.674.91	0.11735	23.179.00	
4.25	0.428 78	20.906.28	4.445 91	4.099 19	0.112.56	29.722.86	
4.30	0.427.67	21.405.04	4.532.22	4,722.86	0.308.00	24,273 16	
4.35	0.426.59	21.909.64	4.616-49	4.74595	0.103 64 0.009 48	24.829.90 25.393.08	
4.40	0.425 54 0.424 53	22.420.06 22.956.33	4,791 73 4,787 93	4,768.48 4,790.45	0.099.48	25.993.08	
443	0.424.53 0.423.55	22.996.33 23.458.42	4,187510	4,790-40	0.093 50	25.992 70 26.138 76	
4.50	0.423 55	24,520 12	5.052 34	4.853.22	0.064.59	27.710-22	
4.70	0.419.92	25.60514	5,233.46	4,892,59	0.078.08	28.907.45	
4.80	0.418.26	26.713.51	5.418.45	4.99011	0.07214	30.130-45	
4.90	0.406.70	27.845.20	5.607.30	4.965.88	0.06670	31.379.21	
5.00	0.415.23	29.000 23	5,800-04	5.000-011	0.06172	32.653 73	

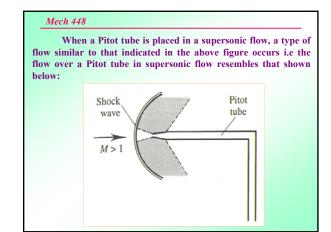




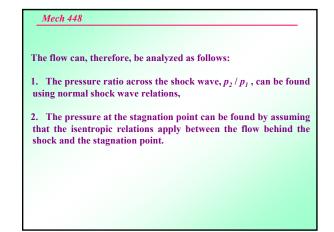
The shock wave ahead of the body is curved in general but ahead of the very front of the body, the shock is effectively normal to the flow. Hence, the conditions across the shock, i.e., between points 1 and 2 in the above figure, are related by the normal shock relations. Further, since the flow downstream of a normal shock wave is always subsonic, the deceleration from point 2 in the figure to point 3 in this figure, where the velocity is effectively zero can, as discussed in the previous chapter, be assumed to be an isentropic process. Using this model of the flow, the pressure at the stagnation pressure can be calculated for any specified upstream conditions.

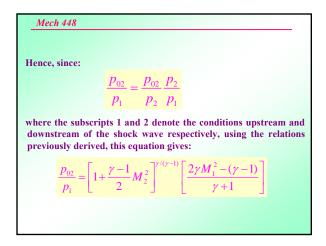
### The flow model is thus:

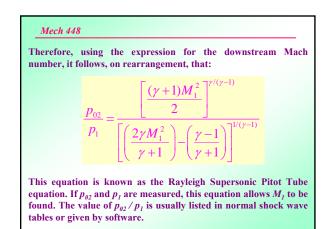
Ahead of 1	-	Undisturbed Flow
1 to 2	-	Normal Shock Wave
2 to 3	-	Isentropic Deceleration to M=0

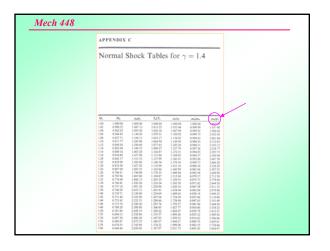


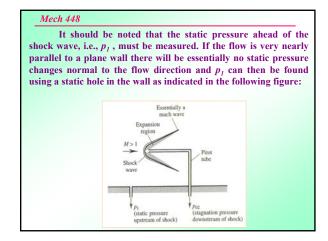
#### Mech 448 Since there will be a change in stagnation pressure across the shock wave, it is not possible to use the subsonic pitot tube equation in supersonic flow. However, as noted above, over the small area of the flow covered by the pressure tap in the nose of the pitot tube the shock wave is effectively normal and the flow behind this portion of the shock wave is, therefore, subsonic and the deceleration isentropic, these assumptions being shown in the following figure. Normal shock shock wave $M_2 < 1$ $M_1 > 1$ M = 0-//-Isentropic deceleration (1)(2)



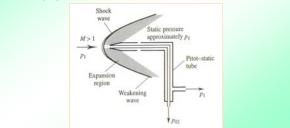


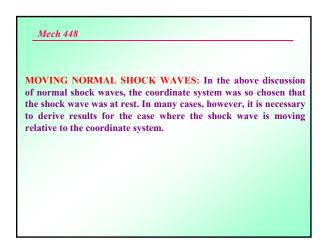


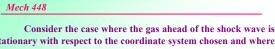




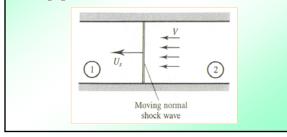
However, it has been found that a Pitot-static tube can also be used in supersonic flow because the shock wave interacts with the expansion waves (see later) decaying rapidly to a Mach wave and the pressure downstream of the vicinity of the nose of the Pitot tube is thus essentially equal to  $p_1$  again as indicated in the following figure.







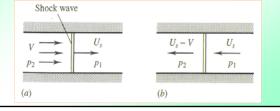
stationary with respect to the coordinate system chosen and where the normal shock wave is moving into this stationary gas inducing a velocity in the direction of shock motion as indicated in the following figure.



# **Mech 448**

Such moving shock waves occur, for example, in the inlet and exhaust systems of I.C. engines, in air-compressors, as the result of explosions and in pipe-lines following the opening or closing of a valve. The required results can be obtained from those that were

derived above for a stationary normal shock wave by noting that the velocities relative to a coordinate system fixed to the shock wave are as indicated in the following figure.



Mech 448Hence, it follows that:
$$V_1 = U_s$$
,  $V_2 = U_s - V$ Since the direction of the flow is obvious, only the magnitudes of  
the velocities will be considered here. The Mach numbers  
upstream and downstream of the shock wave relative to the shock  
wave are given by: $M_1 = \frac{U_s}{a_1} = M_s$  $M_2 = \frac{U_s}{a_2} - \frac{V}{a_2} = \frac{U_s}{a_1} \frac{a_1}{a_2} - \frac{V}{a_2} = M_s \frac{a_1}{a_2} - \frac{V}{a_2} = M_s \frac{a_1}{a_2} - M_2^*$ where: $M_s = \frac{U_s}{a_1}$  and  $M_2' = \frac{V}{a_2}$ 

Normal shock wave tables and software can be used to evaluate the properties of a moving normal shock wave. To do this,  $M_i$  is set equal to  $M_s$  and the tables or software are then used to directly find the pressure, density and temperature ratios across the moving shock wave. Further, since:

$$M'_{2} = M_{s} \frac{a_{1}}{a_{2}} - M_{2}$$

and since  $M_2$  is given by the normal shock tables or software,  $M_2'$ , can be found.

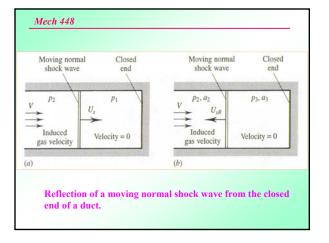
Also since:

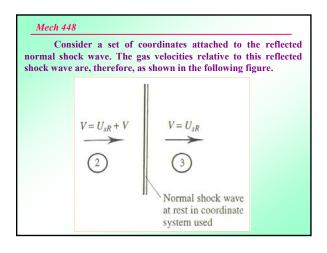
$$\frac{V}{a_1} = M_2 \frac{a_2}{a_1} = M_s - M_2 \frac{a_2}{a_1}$$

V can also thus be deduced using normal shock tables or software.

**Mech 448** 

Brief consideration will now be given to the "reflection" of a moving shock wave off the closed end of a duct. Consider a moving normal shock wave propagating into a gas at rest in a duct. The shock, as discussed above, induces a flow behind it in the direction of shock motion. If the end of the duct is closed, however, there can be no flow out of the duct, i.e., the velocity of the gas in contact with the closed end must always be zero. Therefore, a normal shock wave must be "reflected" off the closed end, the strength of this "reflected" shock wave being just sufficient to reduce the velocity to zero. This is illustrated in the following figure.





Mech 448  
Now:  

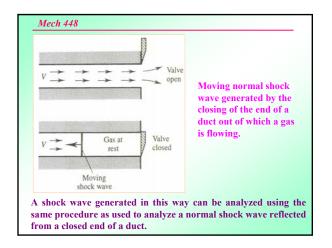
$$M_{R1} = \frac{U_{SR} + V}{a_2} = M_{SR} + M_2'$$

$$M_{R2} = \frac{U_{SR}}{a_3} = \left(\frac{U_{SR}}{a_2}\right) \left(\frac{a_2}{a_3}\right) = M_{SR} \left(\frac{a_2}{a_3}\right)$$
These equations can be used in conjunction with the normal

These equations can be used in conjunction with the normal shock relations previously given or shock tables or the software provided to find the properties of the reflected shock.

### **Mech 448**

Lastly, consider what happens if a gas is flowing out of a duct at a steady rate when the end of the duct is suddenly closed. Since the velocity of the gas in contact with the closed end must again be zero, a shock wave is generated that moves into the moving gas bringing it to rest, i.e., the strength of the shock wave must be such that the velocity reduced to zero behind it. This is illustrated in the following figure.



## **Mech 448**

## **CONCLUDING REMARKS:**

A normal shock wave is an extremely thin region at right angles to the flow across which large changes in the flow variables can occur. Although the flow within the shock wave is complex, it was shown that expressions for the overall changes across the shock can be relatively easily derived. It was also shown that entropy considerations indicate that only compressive shock waves, i.e., shock waves across which the pressure increases, can occur and that the flow ahead of the shock must be supersonic. It was also shown that the flow downstream of a normal shock wave is always subsonic. The analysis of normal shock waves that are moving through a gas was also discussed.

\*

Queens