Parametric Analysis of Turbulent Wall Jet in Still Air Over a Transitional Rough, With Asymptotes of Fully Rough and Fully Smooth Wall Jets

The novel scalings for streamwise variations of the flow in a turbulent wall jet over a fully smooth, transitional, and fully rough surfaces have been analyzed. The universal scaling for arbitrary wall roughness is considered in terms of the roughness friction Reynolds number (that arises from the stream wise variations of roughness in the flow direction) and roughness Reynolds number at the nozzle jet exit. The transitional rough wall jet functional forms have been proposed, whose numerical constants power law index and prefactor are estimated from best fit to the data for several variables, like, maximum wall jet velocity, boundary layer thickness at maxima of wall jet velocity, the jet half width, the friction factor and momentum integral, which are supported by the experimental data. The data shows that the two asymptotes of fully rough and fully smooth surfaces are co-linear with transitional rough surface, predicting same constants for any variable of flow for full smooth, fully rough and transitional rough surfaces. There is no universality of scalings in terms of traditional variables as different expressions are needed for each stage of the transitional roughness. The experimental data provides very good support to our universal relations. [DOI: 10.1115/1.4025005]

Keywords: turbulent wall jet over rough surface, similarity analysis over rough surface, wall bounded turbulent flows, roughness Reynolds number of flow, roughness Reynolds number at nozzle exit

1 Introduction

The turbulent wall jet over a rough surface is of great importance, but is much more poorly understood than smooth wall flow. The technological importance of rough wall bounded turbulent flows is well known. In many situations, a turbulent flow develops over surfaces that are hydro-dynamically rough for some portion of their length. The major impact of wall roughness is to perturb the wall layer which, in general, leads to an increase in wall shear stress that causes erosion of fluid in open channel flow. Further, increase in wall shear stress is almost invariably accompanied by an increase in the wall heat and mass transfer rate. For flows over rough boundaries, the roughness elements prevent the establishment of a viscous boundary layer near the wall. The turbulent viscosity based on roughness length is the relevant parameter which might indicate how the no-slip boundary condition is enforced due to fluid viscosity which ultimately influences the flow.

Rajaratnam [1] conducted research on the wall jet flows over surfaces with deterministic roughness patterns extending from the nozzle exit. Most of the roughnesses used were woven wire meshes or two-dimensional ridges of different types. The thickness of the wire meshes and ridges varied from 1.016 to 11.05 mm (ranging from 2% of the nozzle exit height to almost 53% of the nozzle height). The initial jet Reynolds number ranged from 19,000 to 100,000 and the measurement locations ranged from 2 to 60 times the nozzle height downstream of the nozzle exit.

Rajaratnam [1] showed that the presence of roughness increases the boundary layer thickness and increases its rate of change with $x$. He also showed that while the maximum velocity for a smooth wall jet flow occurs at a $y/d$ of 0.16, the maximum occurs at $y/d$ equal to anywhere from 0.25 to 0.40 for rough wall jet flows depending on the roughness. Here, $\delta$ is the thickness of wall jet where velocity is $U_m/2$ and $U_m$ is the maximum velocity of turbulent wall jet as shown in Fig. 1. The maximum velocity varies with the roughness height as

$$U_m/U_o = C - 0.52 \ln(x/\delta_e),$$

where $\delta_e$ is the melted down roughness height and $C$ is an

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empirically determined constant that varies with the roughness condition. Experimental results of Rajaratnam [1] were presented by Hogg et al. [2] demonstrating the ability to scale the boundary layer thickness on the roughness size \( k_s \), where as smooth wall case considered by Narasimha et al. [3].

Hogg et al. [2] in a turbulent wall jet over a fully rough surface considered the stream wise variation subjected to initial momentum flux \( J = bU_0^2 \) and a measure of turbulent kinematic viscosity \( \nu_t \). Strictly this eddy viscosity is given by \( \nu_t = u_kc_0 \) where \( u_k \) is the friction velocity and \( k_0 \) is the roughness length proportional to grain size. However, there is no initial measure of \( u_k \) and so Hogg et al. [2] used \( U_0 \) instead, which implies \( \nu_t = U_0c_0 \) over a rough wall. It is more appropriate, as shown in present work, to imply that \( u_k = c_0U_0 \) and \( \nu_t = U_0c_0 \), where constant \( c \) may arise due to manner of defining wall roughness parameter.

A turbulent wall jet over a transitional rough surface has been considered by Afzal and Seena [4]. Based on the partial differential equations for mass and momentum subjected to initial momentum \( J \) at the slot, the universal log laws and power law scalings were proposed for velocity, skin friction, and Reynolds shear stress in terms of the roughness Reynolds number and roughness friction Reynolds number. The two asymptotes of log law as well as power law theories correspond to flow over a surface which are fully smooth (Afzal [5]) and fully rough (Afzal and Seena [4]).

Here, we analyze the recent experimental data of Smith [6] in which the coordinate system origin is placed at the nozzle exit (junction between the aluminum plate and the nozzle) on the plate surface at the spanwise center of the nozzle. The streamwise dimension, \( x \), is measured from the origin and increases with downstream distance from the nozzle exit. The vertical dimension, \( y \), is measured positive from the plate surface, and the spanwise dimension, \( z \), is measured from the plate spanwise centerline of the flow facility. For smooth wall test cases the main parameters of data are tabulated in Table 3.1 on page 79 of Smith [6] and for rough wall test cases the main parameters of data is tabulated in Table 3.2, on page 81 of Smith [6], which are divided into test conditions designated A through E with same initial flow conditions for the smooth as well as rough wall test cases. The each test condition represents a particular set of initial flow conditions, namely, the nozzle exit velocity and the nozzle height. The roughness patch grid designation is given for each of the measurements are presented. This designation ranges from 20 to 220 in the current study and is nothing more than a measure of how big the roughness grains are on the sand paper patches used in this experiment. The higher the grid number, the smaller the grain size. The values of \( k_s \) presented are the nominal sizes of the sand grains obtained from standardized tables. The values of \( k_s \) presented are the calculated Nikuradse equivalent sand grain roughness sizes for each of the rough surfaces presented. The equivalent sand grain roughness size is determined empirically by fitting the semilog region of the wall jet flow over rough surfaces to account for the differences in the wall jet semilog region behavior with respect to that of a turbulent boundary layer. In Table 3.2, for test conditions designated A through E are tabulated in Smith [6] in terms of the jet initial velocity \( U_0 \), friction velocity \( U_\infty \), slot width \( b \), jet half width \( d_s \), and friction velocity \( U_\infty \) in normal distance measured from wall to the location where mean velocity decreases to \( U_0 \) in the outer flow). Here, \( U_m \) is the maximum velocity in the wall jet which occurs at a distance \( \delta_m \) from the wall. The outer edge of wall jet boundary layer is located at a distance \( \delta_m \) from the wall where nondimensional velocity \( u/\delta_m \) is one percent or so. The Karman momentum integral, from boundary layer Eqs. (1) and (2), becomes

\[
\frac{dM}{dx} = -\frac{\tau_w}{\rho}, \quad M = \int_0^\infty u^2 dy = \int_0^\infty \frac{\tau_w}{\rho} dx
\]

Let \( \tau_w = \mu \frac{\partial u}{\partial y} \) be the friction velocity, \( C_f = \frac{2\tau_w}{\rho U_\infty^2} \) is coefficient of skin friction and \( \tau_w = \mu \frac{\partial u}{\partial y} \) is the wall shear stress. The nozzle Reynolds number is \( Re = U_0b/\nu \), wall jet boundary layer Reynolds number is \( R_t = U_0\delta/\nu \), and friction Reynolds number is \( R_f = U_0\delta/\nu \). Further, \( R_n = U_0\delta_n/\nu \) the Reynolds numbers and \( R_n = U_0\delta_n/\nu \) the friction Reynolds number are based on \( \delta_n \), the distance of maximum velocity from the wall. In a wall jet in still air, the main parameter is \( J \), the initial jet momentum flux at slot. This would have to be obtained as the integral of across the slot at the exit, and can be expressed (in kinematic units) as \( 3U_0U_\infty b \), where \( \alpha \) is a coefficient which must be \( \alpha = 1 \) if (as is common) \( U_0 \) is the maximum velocity at the exit plane as inferred from pressure measurements. In the present analysis the value of \( \alpha \) is rarely available from experiments; but if \( Re \) is sufficiently high and the jet nozzle is favorably designed for

2 Analysis of Turbulent Wall Jet Over Transitional Rough Surface

The two dimensional turbulent boundary layer equations, with constant pressure, on a flat surface are

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]

\[
\frac{\partial u}{\partial x} + \frac{\partial (uv)}{\partial y} = \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial x}
\]

The novel scalings for stream wise variations of the flow of a turbulent wall jet was also considered by Afzal and Seena [4] for a fully smooth surface in the variables of Narasimha et al. [3] and fully rough surfaces in the variables of Hogg et al. [2]. In the present work, the novel scalings for stream wise flow variation of a turbulent wall jet over a transitional rough surface have been analyzed, as universality of scalings in terms of a parameter termed as roughness Reynolds number due to stream wise variations of the flow. The two asymptotes correspond to flow over fully smooth and fully rough surfaces. The best fit relations for maximum wall jet velocity, boundary layer thickness at maxima of wall jet velocity, the jet half width, the friction factor, and momentum integral are supported by the experimental data. There is no universality of scalings in terms of traditional variables as different expressions are needed for each stage of the transitional roughness. The experimental data of Smith [6] is mainly considered which provides very good support to our universal relations proposed in terms of alternate new variables.

\[
M = \int_0^\infty u^2 dy = J - \int_0^\infty \frac{\tau_w}{\rho} dx
\]
Hogg et al. [2] considered the roughness length given by

\[ \frac{\nu}{u^*} = \exp(\kappa \Delta U_g) \]

Fig. 1. On the rough surface, the normal coordinate


\[ \phi = k_x \exp(\kappa(B - B_T)), \quad \Delta U_g = \kappa^{-1} \ln k_x + B - B_T, \]

\[ k_x = \frac{u_* k_x}{\nu} \]

Afzal et al. [7,9] have proposed expressions of \( \phi \) for inflectional surface roughness as

\[ \phi = 1 + \chi_s k_x \exp(\alpha(k_x/\beta)), \quad \phi = 1 + \chi_m k_x [1 - \exp(-k_x/\beta)] \]

and for commercial steel roughness expressions for roughness scale \( \phi \) is described by Afzal [10].

The Colebrook [11] monotonic roughness scale \( \Delta U_g = \kappa^{-1} \ln(1 + \chi_m k_x) \) leads to

\[ \phi = 1 + \chi_s k_x \]

which from Eq. (7) corresponds to \( \alpha = \beta^{-1} = 0 \). Hogg et al. [2] proposed that the roughness length is proportional to the grain size. If a boundary is artificially roughened by particles of a given diameter, then the bed roughness, \( k_x \), which is the length scale of protrusions into the flow, is simply proportional to the particle diameter. Albayraki et al. [12] considered uniformly graded sand of mean diameter \( d_{50} = 2 \text{ mm} \) that is equal to the roughness length \( k_x \) used in the theoretical analysis. Thus, a direct comparison of results arising from various definitions of wall roughness, unless the roughness by three methods are directly correlated, possibly through parameter \( c \).

\[ \frac{U_{m} \nu}{b} \equiv G_m(\xi) = A_m^{-m}, \quad \frac{\delta}{b} = A_\delta Re^{-m} Re_i^{n-2} \]

where \( Re = b U_0/\nu \) is the Reynolds number at the exit of jet nozzle and \( R_f = x U_0/\nu \) is the local Reynolds number of the flow in a wall jet.

The relations for wall shear stress \( \tau_w \) and friction velocity \( u_\tau \) are given by

\[ \frac{\tau_w}{\rho} \left( \frac{\nu}{\nu} \right)^{2} \equiv G_\tau(\zeta) = A_\tau^{m}, \quad \frac{u_\tau}{U_m} = \frac{\sqrt{A_m}}{A_\delta} \left( \frac{m+2}{2} \right) \]

\[ p = -2m + n - 1 \]

where \( Re = \beta U_0/\nu \) is the Reynolds number based on \( x \). These expressions are as follows:

\[ \frac{U_{m}}{U_0} = A_\m Re^{-m} Re_i^{m-2} Re_i^{n-2} \]

and \( R_f = A_\m \sqrt{A_\delta^{m+2}/2} \). The power indices and prefactors \( (m, A_m), (n, A_n), \) and \( (p, A_p) \) for wall jet over a smooth flat surface variables have been estimated from the experimental data by various authors [3,13-17].

The momentum integral Eq. (3) becomes

\[ \frac{d M}{d \xi} = -\frac{\tau_w}{\rho} \left( \frac{\nu}{\nu} \right)^{2} \]

\[ M = \delta U_{m}^{2} I_1, \quad I_1 = \int_{0}^{\infty} \left( \frac{U_m}{\nu} \right)^{2} d\zeta \]

The momentum integral, Eq. (15), becomes

\[ \frac{M}{J} = 1 - \int_{0}^{\infty} \frac{\tau_w}{\rho} \left( \frac{\nu}{\nu} \right)^{2} d\zeta \]
and based on wall shear stress expression, Eq. (13), we get

\[
\frac{M}{J} = 1 - \frac{A_t}{1 + p} \eta^{1+p}, \quad 1 + p < 0
\]  

\[p = -2m + n - 1, \quad A_t = -p(-2m + n)A^2_A d_i\]  

Equations (11) and (13) may also be expressed as

\[
\frac{U_m}{U_0} \sqrt{\frac{x}{b}} = A_u \eta^{m+1/2}, \quad \frac{\delta}{x} = A_u \eta^{-1}, \quad \frac{U_m}{U_0} \sqrt{\frac{\delta}{b}} = A_u \sqrt{A_u} \eta^{n/2-m}
\]

\[
\frac{\tau_w}{\rho U_0^2} \left( \frac{x}{b} \right) = A_i \eta^{p+1}, \quad \zeta = \left( \frac{bU_0}{\nu} \right)^2
\]

The mean turbulent flow variables over a transitional rough wall may be obtained from fully smooth wall variables, according to a principle laid down by Afzal [7] and Afzal and Seena [4] where the smooth wall variables, say, \( y \), \( R_e \), and \( R_n \), are replaced by appropriate transitional rough wall variables; say, \( \zeta = \zeta / \phi \) and \( R_\phi = R_e / \phi \), where \( \phi = \exp(\kappa \Delta U/m) \) is the roughness scale and \( z = y + c \) is the displaced normal co-ordinate due to roughness. An application of this principle to the predictions of fully smooth wall that can be extended to transitional rough wall, where local Reynolds number \( \zeta \) and \( R_e \) over smooth surface are replaced by \( \eta = \zeta / \phi^2 \) and \( R_\phi = R_e / \phi \) the local Reynolds number over a transitional rough surface. Thus, in present work, of parametric analysis of turbulent wall jet in still air, the main streamwise variable is \( \eta = \zeta / \phi^2 \), where \( \zeta = \kappa \eta / \nu^2 \) is nondimensional smooth wall variable of Narasimha et al. [3] divided by smooth wall shear stress expression, Eq.(13), we get

\[
\frac{U_m}{U_0} = A_u \left( \frac{Re}{\phi} \right)^{1-2m} \left( \frac{x}{b} \right)^{-m}
\]

\[
\eta = \frac{x}{b} \left( \frac{Re}{\phi} \right)^{2p+2} \left( \frac{x}{b} \right)^{n}
\]

\[
M / J = 1 - \frac{A_t}{1 + p} \eta^{1+p}, \quad 1 + p < 0
\]

The relations (24a), (24b), and (24c) may also be expressed as

\[
\frac{U_m}{U_0} = A_u \left( \frac{Re}{\phi} \right)^{1-2m} \left( \frac{x}{b} \right)^{-m}
\]

\[
\frac{\delta}{b} = A_d \left( \frac{Re}{\phi} \right)^{2n-2} \left( \frac{x}{b} \right)^n
\]

\[
\frac{\tau_w}{\rho U_0^2} = A_i \left( \frac{Re}{\phi} \right)^{2p+2} \left( \frac{x}{b} \right)^{p}
\]

where \( R_\phi = Re / \phi \) is the friction Reynolds number at exit of jet from the slot. Let \( R_\phi = Re / \phi \) be the roughness Reynolds number of the wall jet flow, which from Eqs. (26a), (26b), and (27) may be expressed as

\[
\frac{U_m}{U_0} = A_r \left( \frac{Re}{\phi} \right)^{1-2m} \left( \frac{x}{b} \right)^{-m}
\]

\[
\frac{\delta}{b} = A_d \left( \frac{Re}{\phi} \right)^{2n-2} \left( \frac{x}{b} \right)^n
\]

\[
\frac{\tau_w}{\rho U_0^2} = A_i \left( \frac{Re}{\phi} \right)^{2p+2} \left( \frac{x}{b} \right)^{p}
\]

3.2 Approximation to Colebrook Expression. Then Eq. (30), in the light of Eq. (10) becomes

\[
\zeta = \frac{x}{b} \left( \frac{Re}{1+cRe} \right)^2
\]

where \( \eta \) is replaced by \( \zeta \). Equations (31) and (32) in new variable, Eq. (35), become
Present power laws for fully smooth wall jet: $f_5 = A_5 \zeta \psi, \zeta = \psi \left( \frac{Re}{\phi} \right)^{2}, \psi = \frac{U_{w0} d}{b}, \phi = 1 + R_l, R_l = \frac{U_{w0} d}{b}$

Our work

\[ \frac{U_w}{U_0} = A_5 \zeta \psi, \frac{\delta}{b} = \frac{\psi}{\phi}, \frac{\tau_w}{\rho U_0^2} = A_5 \zeta \psi \psi \]

Present power laws for fully rough wall jet: $f_5 = A_5 \zeta \psi, \zeta = \frac{x}{b}, \psi = \frac{\delta b}{k^2}, \phi = \frac{U_{w0} d}{b}, \frac{\tau_w}{\rho U_0^2} = A_5 \zeta \psi \psi$

Our work

\[ \frac{U_{w0}}{U_0} = A_5 \zeta \psi, \frac{\delta b}{k^2} = A_5 \zeta \psi, \frac{\tau_w}{\rho U_0^2} = A_5 \zeta \psi \psi \]

In present power laws, the power index and prefactor numerical values for all types of roughness (fully smooth, transitional, fully rough walls) S-TR-R WJ are same; thus are universal numbers for the estimates of the inflectional roughness data of Smith [6]

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The data for $k_t$, the nominal roughness size, $k_t$, the Nikuradse equivalent sand grain roughness height and $k_{ms}$ the RMS roughness height for various grits, tabulated by Smith [6] has been analyzed. Figure 2 shows the nondimensional number, $f_x$ the Reynolds number based on $k_t$ and $u_t$, $k^+$, roughness Reynolds number based on $k_t$ and $u_t$, and $k_{ms}$ the roughness Reynolds number based on $k_{ms}$ and $u_t$ against wall variable $\zeta$. The definition of $\zeta$ shows that for the large Reynolds number, the flow corresponds to fully smooth boundary for large values on $\zeta$, transitional rough boundary for moderate value of $\zeta$ and fully rough wall corresponds to the further lower values of $\zeta$.

The power indices and prefactors ($m$, $A_n$, $n$, $A_d$) and ($p$, $A_r$) from a wall jet over transitional rough surface data of Smith [6] and numerical power index and prefactor for inflectional surface roughness data are given in Table 1. The fully smooth wall variables by [3, 13, 15–17, 19] have also been presented for ready reference in Table 2. The fully rough wall variables considered by various authors ([2, 6, 13]) based on their surface roughness criteria is shown in Table 2.

The transitional rough wall jet data of Smith [6] is shown in Fig. 3. The data represents monotonic roughness (Colebrook 11), and is represented by a straight line $\phi = 1 + \phi k_u$ with $\phi = 0.554$ by trend line fits with $R^2 = 0.98$. The wall jet data on rough surface by Tachie et al. [13] is also shown in the same figure is represented by $\phi = 0.076$ and the trend line fit predicts $R^2 = 0.45$. The roughness function $\Delta U_m$ from the data of Smith [6] and Tachie et al. [14], Rostamy et al. [15] and Banyassady and Piomelli [18] for the turbulent wall jet on a rough flat surface, and proposed monotonic lines for Smith data and Colebrook type roughness.

We now present the experimental data for $U_m$ in various nondimensional forms proposed here for the transitional rough surface, with $J$ determined from $U_m^2b$ so that the effectiveness of either of these choice may be compared. The data of Smith [6] for turbulent wall jet over smooth and rough flat surfaces has been analyzed in terms of various nondimensional variables forms for $U_m$, $\delta_m$, $\delta$, $\delta^+$, $\delta^*$, $\theta$, and $\tau_w$ in terms of the roughness Reynolds number based on $k_t$ and $u_t$. The prediction of $U_m$, the maximum velocity in dimensional stream wise coordinate $\zeta$ is given by universal relation

$$M = 1 - \frac{A_t}{1 + p} e^{-(p+1)\left(\frac{b}{k_t}\right)^{2p+2}} \left(\frac{\zeta^n}{b}\right)^{1+p}$$  \hspace{1cm} (46b)

Fig. 3. The roughness scale $\phi$ versus $k_u$ from data of Smith [6], Tachie et al. [14], Rostamy et al. [15] and Banyassady and Piomelli [18] for the turbulent wall jet on a rough flat surface, and proposed monotonic lines for Smith data and Colebrook type roughness.
Smith [6] data for rough and smooth plain surface wall jet is shown in Fig. 4(a). The data for fully smooth boundary corresponds to the large values of $\zeta$, and fully rough wall for the lower values of $\zeta$ are found to be collinear, as shown in Fig. 4(a). A least square linear fit to data by Eq. (37) yields $m = 0.507$ and $A_v = 4.211$, which represents the transitional rough surface. The fully rough wall correlation of Hogg et al. [2] is also shown in the same figure. The line marked UR-NS is the universal relation based on co-relation proposed by Narasimha et al. [3] for smooth wall constants. The line marked UR-WS is the universal relation based on correlation of Wygnanski et al. [13] for smooth wall constants, and line marked UR-RR is due to Rostamy et al. rough wall based on smooth wall relationship constants.

\[
\frac{U_m}{U_0} \frac{\phi}{Re} = f_{m}^{\phi}(\zeta) = A_{\phi} \zeta^{-m} \tag{47}
\]

The prediction for $U_m$ versus $\delta$ may be expressed as

\[
\frac{U_m}{U_0} \frac{\phi}{Re} = f_{\phi}^{\phi}(\zeta) = A_{\phi} \zeta^{-m} \tag{48a}
\]

\[
\frac{\phi}{\delta} = \frac{(Re)}{\delta} = \frac{1}{\zeta} \tag{48b}
\]

is shown in Fig. 4(b) from fully smooth and fully rough wall jet data of Smith [6]. The data for fully smooth boundary corresponds to the large values of $\zeta$, and fully rough wall for the lower values of $\zeta$. A least square linear fit to data yields $m_3 = -m/n = 0.499$ and $A_\phi = A_\phi \zeta^{-m/3} = 1.14$, which represents transitional rough boundary for the moderate value of $\zeta$. In the absence of a wall the parameter $\nu$ is not significant and roughness scale $\phi$ may be expected to be irrelevant, and Eq. (48) reduce to the well known relations $U_m \propto x^{-1/2}$ and $\delta \propto x$ for a free jet.

The location of the wall jet velocity maxima $\delta_m$ and half width of the wall jet $\delta$ at $u = U_m/2$ are predicted by the following universal relation for transitional rough surface

\[
\frac{\delta_m}{\delta} \left( \frac{Re}{\delta} \right)^2 = f_{\delta_m}(\zeta) = A_{\delta} \zeta^{n_{\delta_m}} \tag{49a}
\]
The data of Smith [6] for fully smooth and fully rough wall jet shown in Figs. 5(a) and 5(b), respectively, are co-linear and a least square linear fit to data yields \((\alpha) = 0.994, A_\alpha = 0.032\) and momentum thickness data \((\eta_\beta = 0.944, A_\eta = 0.00188)\) for transitional rough walls.

The wall shear stress \(\tau_w = \frac{p U_m^2}{\mu} \frac{\phi}{(Re_s)^2} = f_s(\zeta) = A_s \zeta^{-0.032}\) for transitional rough wall. The alternate universal relation on constants \((p = -1.07, A_s = 0.146)\) proposed by Wygnanski [11] from fully smooth wall is also shown in Fig. 7.

The multiplication of relation (48b) with square root of relation (49b) yields

\[
\delta \left( \frac{Re}{\phi} \right)^{1/2} = f_\delta(\zeta) = A_\delta \zeta^{0.063}
\]

where
\[
A_\delta = 0.146
\]

The data of Smith [6] for fully smooth and fully rough wall jet shown in Fig. 7 are co-linear and a least square linear fit to data yields \((\alpha) = 0.994, A_\alpha = 0.032\) and momentum thickness data \((\eta_\beta = 0.944, A_\eta = 0.00188)\) for transitional rough walls.

The wall shear stress \(\tau_w = \frac{p U_m^2}{\mu} \frac{\phi}{(Re_s)^2} = f_s(\zeta) = A_s \zeta^{-0.032}\) for transitional rough walls. The alternate universal relation based on constants \((p = -1.07, A_s = 0.146)\) proposed by Wygnanski [11] from fully smooth wall is also shown in Fig. 7.

The multiplication of relation (48b) with square root of relation (49b) yields

The displacement thickness \(\delta\) and momentum thickness \(\theta\) are predicted by the following universal relations for transitional rough surface

\[
\delta \left( \frac{Re}{\phi} \right)^{1/2} = f_\delta(\zeta) = A_\delta \zeta^{0.063}
\]

\[
\theta \left( \frac{Re}{\phi} \right)^{1/2} = f_\theta(\zeta) = A_\theta \zeta^{0.063}
\]

The data of Smith [6] for fully smooth and fully rough wall jet shown in Figs. 6(a) and 6(b), respectively, are co-linear

\[
\delta(Re/\phi)^{1/2} = f_\delta(\zeta) = A_\delta \zeta^{0.063}
\]

\[
\theta(Re/\phi)^{1/2} = f_\theta(\zeta) = A_\theta \zeta^{0.063}
\]

yields. The data of Smith [6] for fully smooth and fully rough wall jet shown, in Figs. 6(a) and 6(b), respectively, are co-linear

\[
\delta(Re/\phi)^{1/2} = f_\delta(\zeta) = A_\delta \zeta^{0.063}
\]

\[
\theta(Re/\phi)^{1/2} = f_\theta(\zeta) = A_\theta \zeta^{0.063}
\]
which is also tested from data. In Eq (52) left hand side is the ratio $U_{sl}/U_{0}$ of wall jet maximum velocity $U_{sl}$ to exit velocity of jet slot $U_{0}$ multiplied with square root of the ratio of wall jet half-thickness $\delta$ to exit of jet slot exit $b$ is represented by power law relation. The turbulent wall jet over a fully smooth and fully rough data of Smith [6] are shown here in Fig 8. The proposed universal relation for transitional rough surface shown in Fig 8 is $y = (U_{sl}/U_{0})(\delta/b)^{1/2} = 1.3 x^{0.01}$, where $x = \zeta$; have been co-linear with fully rough and fully smooth wall jet data.

5 Conclusions

(1) The novel streamwise scaling for turbulent wall jet over fully smooth surface has been studied by Narasimha et al. [3], Wygnanski et al. [13], Tachie et al. [17], and Afzal [5], and fully rough surface by Hogg et al. [2], Albayraki [12], and Afzal and Seena [4].

(2) For turbulent wall jet over smooth surface has been demonstrated by data of Tachie et al. [17] and transitionally rough surfaces by Rostamy et al. [16,17]. There is no universality of scalings in terms of traditional variables as different expressions are needed for each stage of the transitional roughness.

(3) The universal novel scalings are proposed here for the stream wise variations of the flow in a turbulent wall jet over a transitional rough surface in terms of a new parameters termed as the roughness friction Reynolds number and the roughness Reynolds number.

(4) The roughness friction Reynolds number is defined as $Re_{\phi} = Re_{\delta}/\phi$ and roughness Reynolds number is defined as $Re_{m0} = Re_{u}/\phi$, where $\phi$ is the roughness scale connected with roughness function. For fully smooth surface $\phi = 1$ and thus, $Re_{\phi} = Re_{\delta}$ and roughness Reynolds number $Re_{m0} = Re_{u}/\phi$. For fully rough surface $\phi = \gamma u_{k}/\nu$ and roughness Reynolds number $Re_{m0} \sim (U_{sl}/U_{0})(\delta/k_{f})$.

(5) For flows over rough boundaries it is not the fluid viscosity which ultimately influences the flow, but the roughness elements which prevent the establishment of a viscous boundary layer. For transitional rough surface $k_{f}$ in present work analyzed the data based on Nikuradse sand grain roughness [6]. On other hand, Hogg et al. [2] and Afzal and Seena [4] considered the roughness length $k_{f}$ proportional to grain size and Albayraki et al. [12] adopted $k_{f} = d_{50} = 2$ mm, where uniformly graded sand of mean diameter $d_{50}$ that is equal to the roughness length $k_{f}$ used in the theoretical analysis. Thus, a direct comparison of results arising from various definitions of wall roughness, unless the roughness by three methods are directly correlated, possibly through parameter $c$, which presently is under consideration of further work.

(6) In the turbulent wall jet over transitional rough, fully rough and fully smooth wall, the functional form for power law has been proposed for (i) maximum wall jet velocity, (ii) boundary layer thickness at maxima of wall jet velocity, (iii) the jet half width, (iv) the friction factor, and (v) momentum integral.

(7) That for each of above power law, the numerical values of power index and prefactor remain same, whether wall is transitional rough, fully rough and fully smooth. These universal numerical values has been estimated from data of Smith [6] for fully smooth, transitional and fully rough for Nikuradse sand grain roughness in the wall jets.

(8) The data shows that the two asymptotes for fully rough surface and fully smooth are co-linear, with transitional rough surface; thus predicting same constants for any variable of flow for the cases of full smooth, fully rough and transitional rough surface considered here.

References