Mathematical modeling of solar chimney power plants

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Abstract

The solar chimney power plant is a system with promise to generate electrical power from free solar energy. In this study, a solar collector, chimney and turbine are modeled together theoretically, and the iteration techniques are carried out to solve the resulting mathematical model. Results are validated by measurements from an actual physical plant. Moreover, the model is employed to predict the performance characteristics of large-scale commercial solar chimneys, indicating that the plant size, the factor of pressure drop at the turbine, and solar heat flux are important parameters for performance enhancement. In addition, the study proposes that the most suitable plant, affordable by local government standards to respond to the electricity demand of a typical village in Thailand, is the one with a collector radius and chimney height of 200 m and 400 m, respectively. Furthermore, it is shown that the optimum ratio between the turbine extraction pressure and the available driving pressure for solar chimney systems is also proposed in the study using dimensional analysis.

1. Introduction

The solar chimney power plant is a solar electricity production facility, which uses solar radiation to increase the internal energy of air flowing through the system. The schematic of a typical solar chimney is sketched in Fig. 1. In this plant, air is heated as a result of the greenhouse effect under a translucent roof (collector). As the roof is open at its periphery, buoyancy drives a continuous flow from the roof perimeter into the chimney located at the middle of the roof. An electricity-generating turbine is set in the path of the air current to convert the kinetic energy of the flowing air into electricity.

Solar chimney power plants can convert only a small portion of the collected solar energy into electricity, but they make up for this disadvantage by their economical, robust construction and low maintenance costs [1]. Economic viability requires the optimum configurations of each component. Efficient conversion of fluid power to shaft power depends primarily on the turbine operation. During the day, mass flow through a system changes, due mainly to the variation of solar radiation. Accordingly, the turbine blade pitch should be adjusted during its operation to regulate the power output. In this study, the optimal operating conditions of the turbine unit are determined in consideration of the plant power output.

To evaluate the available plant power output, researchers defined the ratio of the pressure drop across the turbine to the total available driving pressure of a system, which is symbolized as $P_{turb}/P_{tot}$ herein. Indeed, most investigators have assumed that the optimum $P_{turb}/P_{tot}$ is 2/3 ([1–8]). Although the computations of values taken from Ref. [1] showed that $P_{turb}/P_{tot}$ used is 0.82, Ref. [9] illustrated that the ratio is between 0.66 and 0.7 during the day. Furthermore Von Backström and Fluri [10], stated without reference that the optimum ratio is 0.8. Additionally Nizetic and Klarin, Schlaich et al. [11,12], proposed analytical approaches, and showed that the pressure ratio varied in the range of 0.8–0.9. Meanwhile Bernardes et al. [13] reported an optimum value of 0.97. Although the turbine under consideration is encased by a tower, some authors use the ratio as 16/27 ([14–16]), which is the Betz limit obtained for an actuator disc in a free stream situation.

We develop a detailed theoretical model here to evaluate the performance of solar chimney power systems. Our paper also presents the operating range of the turbine. It tries to determine how to maximize the fluid power by adjusting the pressure drop across the turbine and the flow through it.

2. Optimal pressure ratio

According to the operation principle mentioned above, the air inside a system is less dense than the atmospheric air outside. The air moves in and out of the solar chimney system continuously, driven by the pressure difference between the inside and outside.
This pressure difference will be called the available driving pressure and symbolized as $\Delta p_{tot}$. Neglecting friction losses [17], $\Delta p_{tot}$ can be subdivided into a turbine extraction component representing the pressure extracted at the turbine, and a dynamic component describing the kinetic energy of the airflow:

$$\Delta p_{tot} = \Delta p_{turb} + \Delta p_{dyn}. \quad (1)$$

Let us define the ratio $p_{turb}/p_{tot}$ as $x$, so that

$$\Delta p_{turb} = x\Delta p_{tot}. \quad (2)$$

Using the standard definition for dynamic pressure, we obtain

$$\Delta p_{dyn} = \frac{1}{2} \rho c V_{with\ turb}^2. \quad (3)$$

Without the turbine, the maximum flow speed is achieved and the whole driving potential is used to accelerate the flow, so that

$$\Delta p_{tot} = \frac{1}{2} \rho c V_{no \ turb}^2. \quad (4)$$

Substituting Eqs. (2)–(4) into Eq. (1) yields,

$$V_{with\ turb} = V_{no \ turb} \sqrt{\frac{1}{1 - x}}. \quad (5)$$

The theoretical power extracted by the turbine can be determined from the energy equation and Gibbs relation from classical thermodynamics:

$$W_{ext} = m \int vdp = \frac{m}{\rho_{turb}} \Delta p_{turb}. \quad (6)$$

Substituting Eqs. (2) and (5) into Eq. (6) and using $m = \rho_{turb} A_c V_{with\ turb}$, we obtain:

$$W_{ext} = A_c \cdot \sqrt{1 - x} \cdot V_{no \ turb} \cdot x \cdot \Delta p_{tot}. \quad (7)$$

The optimal $x$ for the maximum power extraction can be obtained by assuming that $V_{no \ turb}$ and $\Delta p_{tot}$ are not functions of $x$ and solving $\partial W_{ext}/\partial x = 0$. The result for the optimal pressure ratio is

$$x_{opt} = \frac{2}{3}. \quad (8)$$

Consequently, maximum power is obtained when the turbine extraction pressure is $2/3$ of the available driving pressure, corresponding to the value that most researchers have utilized. Using our assumptions, the optimal pressure ratio of $2/3$ is valid only for the constant-driving-pressure systems (i.e. for the constant air temperature increase).

Equation (7) shows that the plant power output can be increased by adjusting the turbine extraction pressure. Later, it will be shown that changing $x$ affects $m$, $W_{ext}$, and $\Delta p_{tot}$. As a result, the theoretical power output can be increased by adjusting the turbine extraction pressure.
assumption that $\Delta p_{\text{tot}}$ is not the function of $x$ would render inaccurate the predicted power.

3. The mathematical model

With the stations numbered as in Fig. 1, the temperature rise can be estimated from the energy equation across the roof portion:

$$m c_p(T_2 - T_1) + \frac{1}{2} m (V_2^2 - V_1^2) = q''A_r$$

(9)

For simplicity, frictional effect is ignored since the velocity in this region is quite low. Because the flow is in the very low Mach number regime, the kinetic energy contribution can be safely neglected, therefore,

$$m c_p\Delta T = q''A_r.$$  

(10)

When the inlet solar radiation is assumed constant, Eq. (10) shows that the temperature rise is inversely proportional to the mass flow rate.

In this analysis, the turbine is treated as the Rankine–Froude actuator disc. The assumptions on which this actuator disc based are listed as follows [18]:

1. Steady, homogeneous wind.
2. Uniform flow velocity at disc.
3. Static pressure decreases discontinuously across the disc.
4. No rotation of flow produced by disc.

Accordingly, Eq. (6) becomes,

$$W_{\text{ext}} = \frac{m}{(p_2 + p_3)/2} (p_2 - p_3).$$

(11)

By synthesizing equations for continuity, momentum and energy of the flow under the roof Chitsomboon [19], proposed that

$$p_2 - p_1 = \rho V^2 \left( \frac{dA}{A} - q''dA_r / m c_p T \right).$$

(12)

Assuming that $q''$, $c_p$ and $m$ are constant, $\rho$ and $T$ can be approximated to be $\rho_1$ and $T_1$ without significantly affecting the numerical values of the terms. The Mach number is again assumed to be very low and thus can be neglected; the equation is then simplified to be,

Table 1
Geometrical dimensions of the pilot plant in Manzanares, Spain.

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Value (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean roof radius, $r_1$</td>
<td>122</td>
</tr>
<tr>
<td>Average roof height, $h_1$</td>
<td>1.85</td>
</tr>
<tr>
<td>Tower height, $h_2$</td>
<td>194.6</td>
</tr>
<tr>
<td>Tower radius, $r_c$</td>
<td>5.08</td>
</tr>
</tbody>
</table>

Table 2
Comparison between measured data from Manzanares pilot plant and theoretical results. (data on 2nd September 1982 taken from Ref. [23]).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Measured</th>
<th>Theory</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time (h)</td>
<td>10.00</td>
<td>12.00</td>
</tr>
<tr>
<td>$I$ (W/m²)</td>
<td>744.4</td>
<td>850</td>
</tr>
<tr>
<td>$\eta_{\text{meas}}$ (%)</td>
<td>24.3</td>
<td>27.1</td>
</tr>
<tr>
<td>$T_1$ (°C)</td>
<td>21.1</td>
<td>23.4</td>
</tr>
<tr>
<td>$\Delta T_{12}$ (°C)</td>
<td>14.8</td>
<td>17.8</td>
</tr>
<tr>
<td>$V_4$ (m/s)</td>
<td>7.7</td>
<td>7.7</td>
</tr>
<tr>
<td>$\Delta p_{\text{turb}}$ (mbar)</td>
<td>6.29</td>
<td>5.28</td>
</tr>
</tbody>
</table>

Table 3
Data of Manzanares pilot plant for 1st September 1989 taken from Ref. [24].

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Measured</th>
<th>Theory</th>
</tr>
</thead>
<tbody>
<tr>
<td>Global solar radiation (W/m²), $I$</td>
<td>1017</td>
<td></td>
</tr>
<tr>
<td>Ambient temperature (°C), $T_1$</td>
<td>18.5</td>
<td></td>
</tr>
<tr>
<td>Ambient pressure (Pa), $p_1$</td>
<td>929.30</td>
<td></td>
</tr>
<tr>
<td>Collector absorption coefficient, $\alpha$</td>
<td>0.65</td>
<td></td>
</tr>
<tr>
<td>Collector loss coefficient (W/m² K), $U$</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>Turbine efficiency</td>
<td>0.85</td>
<td></td>
</tr>
<tr>
<td>Generator and gearbox efficiency</td>
<td>0.9</td>
<td></td>
</tr>
<tr>
<td>Upwind velocity (m/s), $V_4$</td>
<td>8.1</td>
<td></td>
</tr>
</tbody>
</table>

Table 4
Comparison between measured data from Manzanares pilot plant and theoretical results. (data on 1st September 1989).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Measured</th>
<th>Theory</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_1$ (°C)</td>
<td>38</td>
<td>41.5</td>
</tr>
<tr>
<td>$W_{\text{ext}}$ (kW)</td>
<td>48.4</td>
<td>48.3</td>
</tr>
</tbody>
</table>

$$p_2 = p_1 + \frac{m q''}{2 \pi h_2^2 \rho_1 c_p T_1} \ln \frac{r_c}{r_1} - \frac{m^2}{2 \rho_1} \left( \frac{1}{A_2^2} - \frac{1}{A_1^2} \right)$$

(13)

where $p_1$, $p_2$ and $T_1$ are approximated as $p_a$, $p_a$ and $T_a$, respectively. Eq. (13) shows that $p_2$ is the combination of the inlet pressure, $p_1$, with the pressure increase due to heat addition, $(m q''/2 \pi h_2^2 \rho_1 c_p T_1) \ln r_c/r_1$, and the pressure decrease due to flow area reduction towards the center of the roof, $m^2/2 \rho_1 (1/A_2^2 - 1/A_1^2)$. Order of magnitude analysis reveals that $m^2/2 \rho_1 (1/A_2^2 - 1/A_1^2)$ is significantly greater than $(m q''/2 \pi h_2^2 \rho_1 c_p T_1) \ln r_c/r_1$.

Rearranging Eq. (10) yields

$$T_2 = T_1 + q''A_r / m c_p,$$

(14)

where, according to Ref. [20],

$$q'' = \alpha I - U \Delta T.$$  

(15)

It should be noted that $\Delta T$ in Eq. (15) is the temperature difference between the air inside and outside a solar collector which is assumed here to be equal to $(T_2 - T_1)$.

Accordingly,

$$\Delta T = \frac{\alpha I - U}{m c_p / A_r + U}.$$  

(16)

If the work extraction process at the turbine is assumed to be an isentropic process, then

$$T_3 = T_2 \left( \frac{p_3}{p_2} \right)^{\frac{-1}{\gamma-1}}.$$  

(17)

Furthermore, by rearranging the momentum and continuity equations for the flow through a constant area vertical tower of height, $h_c$, we obtain

$$p_3 = p_4 + \frac{1}{2} (\rho_3 + \rho_4) g h_c + \left( \frac{m}{A_c} \right)^2 \left( \frac{1}{p_4} - \frac{1}{p_3} \right).$$  

(18)

If we consider the atmospheric air outside the solar chimney system, the hydrostatic equilibrium requires that

$$\frac{dp}{dz} = -\rho g.$$  

(19)

According to Ref. [21], when the atmospheric air parcel is regarded as an unsaturated medium and expands slowly to a lower
Fig. 3. Influence of mass flow rate on power output for solar irradiation \(= 600 \text{ W/m}^2\). \((h_t = 2 \text{ m and } r_t = 4 \text{ m for all plants})\).

Fig. 4. Influence of pressure ratio (cf. Eq. (2)) on power output for solar irradiation \(= 600 \text{ W/m}^2\). \((h_t = 2 \text{ m and } r_t = 4 \text{ m for all plants})\).

Fig. 5. Influence of mass flow rate on collector temperature rise for solar irradiation \(= 600 \text{ W/m}^2\). \((h_t = 2 \text{ m and } r_t = 4 \text{ m for all plants})\).
atmospheric pressure without exchange of heat, the dry adiabatic temperature lapse rate equation can be written as

$$T = T_\infty - \frac{g}{c_p} z.$$  \hspace{1cm} \text{(20)}

Assuming that the air obeys the ideal gas equation of state, Eq. (20) can be substituted into Eq. (19) to give $p_4$ for the outside air as

$$p_4 = p_{\infty} \left(1 - \frac{g}{c_p T_\infty} h_c\right).$$  \hspace{1cm} \text{(21)}

Consider that a dry adiabatic lapse rate can be applied to the flow in a tower. In accordance with Eq. (20),

$$T_4 = T_3 - \frac{g}{c_p} h_c.$$  \hspace{1cm} \text{(22)}

For an ideal gas,

$$\rho_2 = \frac{p_2}{RT_2}, \quad \rho_3 = \frac{p_3}{RT_3}, \quad \rho_4 = \frac{p_4}{RT_4}.$$  \hspace{1cm} \text{(23)}

Theoretically the driving pressure of the solar chimney ($\Delta p_{\text{tot}}$) is the difference between pressure potentials caused by the column of cold air outside the chimney and the corresponding column of warm air inside the chimney \cite{22}. Consider that the pressure at the chimney exit and the pressure outside the chimney but at the same height as the chimney exit are approximately equal. Consequently, in this analysis we propose

$$\Delta p_{\text{tot}} = p_1 - p_3.$$  \hspace{1cm} \text{(24)}
One can argue that this way of definition makes $\Delta p_{\text{tot}}$ insensitive to the chimney height. Numerical simulation of several different plant sizes, on the other hand, showed that $\Delta p_{\text{tot}}$, calculated by using Eq. (24), is a function of the chimney height.

4. Solution procedure

According to the formulation above, if the mass flow rate is known or assumed then the power output can be attained. The steps of calculating the plant power output are:

1. Choose the mass flow rate,
2. Calculate $T_2$ using Eq. (16),
3. Calculate $q''$ using Eq. (15),
4. Calculate $p_2$ using Eq. (13),
5. Calculate $p_2$ using Eq. (23),
6. Calculate $p_4$ using Eq. (21),
7. Guess $p_3$, then calculate $T_3$, $p_3$, $T_4$, $p_4$ using Eq. (17), Eq. (23), Eq. (22) and Eq. (23), respectively. Calculate $p_3$ using Eq. (18), then compare the new $p_3$ to the former $p_3$. Perform the iteration process until the difference between corresponding new and old $p_3$ is less than an acceptable value.
8. Calculate $W_{\text{ext}}$ using Eq. (11),
9. Calculate $\Delta p_{\text{tot}}$ using Eq. (24).

A flowchart for these procedures is illustrated in Fig. 2.

5. Results and discussion

5.1. Validation of the model with experimental data

To validate the analytical model, our theoretical data was compared with the experimental results of the prototype from Manzanares, Spain. The plant dimensions are given in Table 1.

The measured data on September 2nd, 1982 are adopted from Ref. [23]. The comparisons between the theoretical predictions and the experimental values are presented in Table 2. Based on the data provided by the reference article, instead of using Eq. (15), $q''$ was computed from
where \( \eta_{\text{coll}} \) is the collector efficiency (defined as in Eq. (26)). The values of \( I, \eta_{\text{coll}}, T_h, \Delta T_{12}, V_4 \) (measured) and \( \Delta P_{\text{turb}} \) (measured) displayed in Table 2 are taken from Ref. [23]. It is clear that all the predicted values are underestimated in Table 2. This is consistent with the "conservative approximation" of \( p_2 \) as stated in Ref. [19]. The acceptable agreement between experimental and theoretical results for \( V_4 \) and \( \Delta P_{\text{turb}} \) can be clearly seen for the whole range of \( I \).

Furthermore, the measured data from Manzanares prototype plant on September 1st, 1989 are taken from Ref. [24]. The data adopted are presented in Table 3. The comparisons shown in Table 4 indicate good agreement between theoretical and experimental results, which should support the reliability of the proposed models.

5.2. Performance characteristics of solar chimney systems

Eight plant sizes are examined and presented their characteristics in Figs. 3–5. Based on the cost model presented in Ref. [1], these plant dimensions are chosen such that the plant can be affordable by the local government of villages in Thailand. Figs. 3, 4 show the calculated power output as a function of the mass flow rate and \( \Delta P_{\text{turb}}/\Delta P_{\text{rot}} \). The parameters \( \alpha = 0.75 \) and \( U = 5 \text{ W/m}^2 \text{ K} \) for Eq. (15) are taken from Ref. [1]. The temperature rise across the collector is presented in Fig. 5. It is observed that for a specified mass flow rate, the temperature rise is a function of roof radius only; it does not depend on the chimney height, a consequence of the specified conditions and the mathematical model defined by Eq. (14). Thus it should not be construed that the temperature rise is actually not a function of the chimney height. It should also be noted that the designed solar heat flux in Figs. 3–5 is 600 W/m².

According to the total maximum demand of electricity and the number of electrified villages in Thailand reported by the Provincial Electricity Authority of Thailand [25], the power demanded by each village is approximately 200 kW. So the appropriate plant, which has a reasonable temperature rise of 20 K (see Fig. 5) and can serve the electricity demand for each village in Thailand (see Fig. 3), is one with a collector radius of 200 m and a chimney height of 400 m. From now on, any observations will be for this plant size.

The variations of the power as a function of the solar heat gain are shown in Figs. 6, 7. In these figures, \( q'' \) (with loss) refers to the systems that the solar heat fluxes were computed from Eq. (15). The name implies that only some part of the solar radiation was absorbed by the systems. On the other hand, \( q' \) (no loss) in the figures represents systems that absorb solar radiation completely which means the collector efficiency equals 100%. It can be seen that the maximum power of systems with loss occur somewhere between the maximum and minimum mass flow rate while the maximum powers for the systems without heat loss occur at the point that offers the minimum mass flow rate. Apparently the optimum pressure ratio depends on the magnitude of the solar heat flux as displayed in Fig. 7. It is not equal to 2/3, but is approximately 0.84.

To investigate further the effect of the plant size on the flow characteristics, Figs. 8–10 show the relationships between \( \Delta P_{13}, \Delta P_{23}, q'' \) and \( in \) for plants with different sizes. It is clear that their relations depend on the plant sizes. Fig. 11 shows the collector efficiency that is defined as

\[
\eta_{\text{col}} = \frac{m c_p (\Delta T_{12})}{q'' A_f}
\]  

Fig. 10. Influence of mass flow rate on useful heat gain extracted from the collector along lines of constant solar irradiation. (\( h_t = 2 \text{ m} \) and \( r_c = 4 \text{ m} \) for all plants).

Fig. 11. Influence of mass flow rate on collector efficiency (cf. Eq. (26)) along lines of constant solar irradiation. (\( h_t = 2 \text{ m} \) and \( r_c = 4 \text{ m} \) for all plants).
that there is no representative value. We observe that the efficiency decreases as the collector size increases, explainable by the fact that when the plant size increases, the temperature rise across the collector increases as depicted in Fig. 5 and the solar heat gain decreases subsequently as defined by Eq. (15). Thus the collector efficiency is inversely proportional to the collector size.

Next, let us define the product of \( \Delta p_{13}{A_cV_2}/(q''A_rghc_\beta/c_p) \) as the ‘Dimensionless \( \Delta p_{13} \)’. The reasons for this naming are that the product is dimensionless and can be interpreted as “\( \Delta p_{13} \) is scaled by \( (q''A_rghc_\beta/c_p)/A_cV_2 \) in the product.” The characteristic of ‘Dimensionless \( \Delta p_{13} \)’ is illustrated in Fig. 12. Although the flow properties between plants are widely scattered as displayed in Figs. 8–11, it is important to notice from Fig. 12 that the values of ‘Dimensionless \( \Delta p_{13} \)’, are approximately equal to unity for all plants. To investigate the reason for the obtained results, let us consider the dimensionless number proposed in Ref. [26]. Reference [26] proposed that, for solar chimney power plants without a turbine work extraction,

\[
\frac{mv^2}{2} = 1. \tag{27}
\]

Assuming that the whole available driving pressure is used to accelerate the air and is thus converted completely into kinetic energy, we have

\[
\Delta p_{13}A_cV_2 = \frac{mv^2}{2}. \tag{28}
\]

Consequently,

\[
\frac{\Delta p_{13}A_cV_2}{q''A_rghc_\beta/c_p} = 1 \tag{29}
\]

as confirmed by Fig. 12. As a result, Eq. (29) can principally be used to evaluate the available driving pressure of solar chimney power plants.

Fig. 13 presents \( \Delta p_{loss} \), the pressure difference between \( \Delta p_{13} \) and \( \Delta p_{23} \). Surprisingly all data collapse into one single line! It is found that

\[
\Delta p_{loss} = 0.0002m^2 \tag{30}
\]

regardless of the plant size or the solar heat flux! Consequently, we can use Eqs. (29) and (30) together with the collector efficiency from Fig. 11 to approximate the turbine output power for solar chimney systems.

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**Fig. 12.** Influence of mass flow rate on dimensionless \( \Delta p_{13} \) (cf. Eq. (29)) along lines of constant solar irradiation. (\( h_r = 2 \) m and \( r_c = 4 \) m for all plants).

**Fig. 13.** Influence of mass flow rate on plant total pressure loss along lines of constant solar irradiation. (\( h_r = 2 \) m and \( r_c = 4 \) m for all plants).
6. Conclusion

Theoretical simulations were conducted in order to evaluate the performance of solar chimney power plants. The relationships between the pressure ratio and the mass flow rate and between the temperature rise across the collector and the power output were presented. It was found that, for the system with a constant driving pressure (available system pressure difference), the optimum ratio of the turbine extraction pressure to the driving pressure is 2/3. For the system with the non-constant driving pressure, it is obvious that this optimum ratio is a function of the plant size and solar heat flux. This observation would be helpful in the preliminary plant design. In addition, it was shown that the appropriate plant, which can serve the electricity demand for each village in Thailand and the investment cost would be affordable by the local government, is the one with a collector radius of 200 m and a chimney height of 400 m. The optimum pressure ratio for the proposed plant is equal to 0.84 approximately. The paper also proposes the simple method to primarily evaluate the turbine power output for solar chimney systems.

Economic viability requires the optimum configurations of each component. To develop a mathematical statement for optimization, we need a simple-but-accurate mathematical model of solar chimney power plants. Even though the solar chimney power plant concept has been proposed and intensively studied before by several different researchers, the mathematical models proposed in the literature are mathematically so complex and some of them do not in the form that suitable for us to optimize. The mathematical model presented here is relatively simple while provides a very accurate result as shown in Table 4. So we will use the present mathematical model to find the optimum operating conditions for the whole system in our next study.

Acknowledgments

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References


Nomenclature

A: flow area, m²
A: roof area, m²
cₚ: specific heat capacity at constant pressure, J/(kg K)
g: gravitational acceleration, m/s²
hₘ: chimney height, m
hₐ: roof height above the ground, m
I: solar irradiation, W/m²
M: Mach number
m: mass flow rate, kg/s
p: pressure, Pa
q*: insolation, W/m²
R: ideal gas constant, J/kg K
rₘ: chimney radius, m
rₖ: roof radius, m
T: absolute temperature, K
U: collector loss coefficient, W/m² K
V: flow velocity, m/s
x: pressure ratio, Eq. (2)
v: specific volume, m³/kg
W: power, W
z: cartesian coordinate in vertical direction
Greek symbols

α: collector absorption coefficient
β: coefficient of volumetric thermal expansion, 1/K
Δp: pressure drop, Pa
ΔT: temperature rise between ambient and collector outlet, °C
γ: specific heat ratio
ηₑ: collector efficiency
ρ: density, kg/m³

Subscripts

c: chimney
dyn: dynamic pressure component
ext: extraction by the turbine
loss: difference between the available driving pressure and the turbine extraction pressure
no turb: without turbine
opt: optimum
r: roof
tot: total pressure component
turb: turbine
with turb: with turbine
=: free stream