Turbulence

(February 15, 2002)

The literature for this lecture (denoted by LD) and the following on turbulence models is:

L. Davidson. *An Introduction to Turbulence Models.* Report 97/2, Dept. of Thermo and Fluid Dynamics, Chalmers, 1997

Turbulence is:

- Irregular
- Diffusive
- Occurs at high Re numbers
- Three-dimensional
- Dissipative
- Contrary to viscosity, turbulence is a feature of the *flow*, not of the *fluid*

Length scales

- Large scales, denoted by

$$\mathcal{U},\ \ell$$

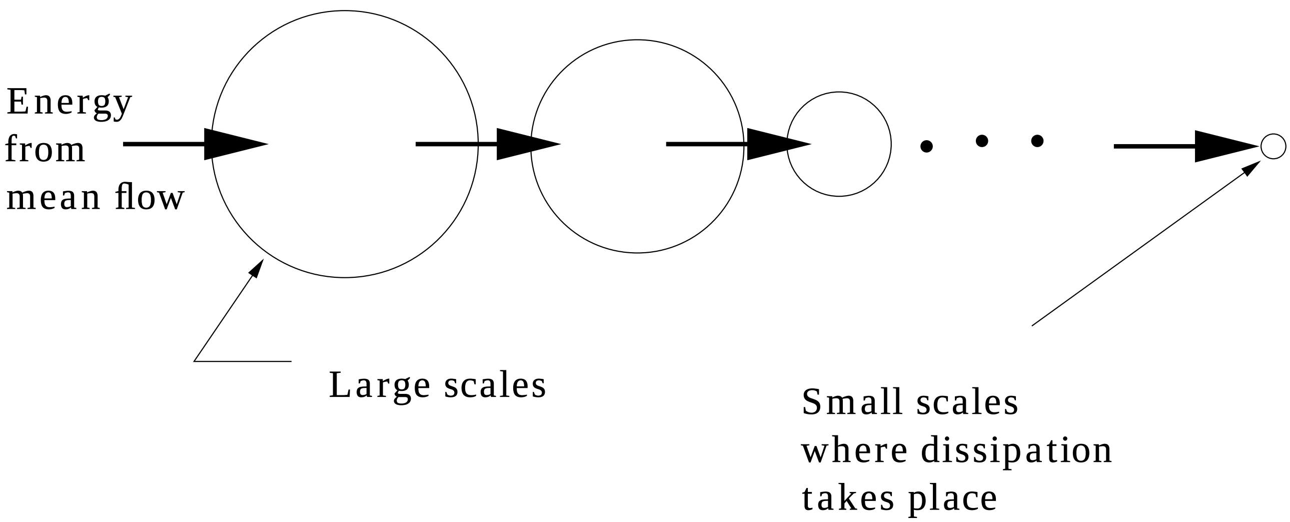
takes energy from the mean flow.

- Smaller scales takes energy from the large scales

$\vdots$

• Smallest scales takes energy from the "next-to-the-smallest" scales. At the smallest scales the kinetic energy is transformed into internal energy (increase in temperature): this is called *dissipation*.

Kinetic energy is extracted from the mean flow by the large eddies; this energy is then transferred via smaller and smaller eddies to the smallest eddies: this is called "the cascade process", see figure below.

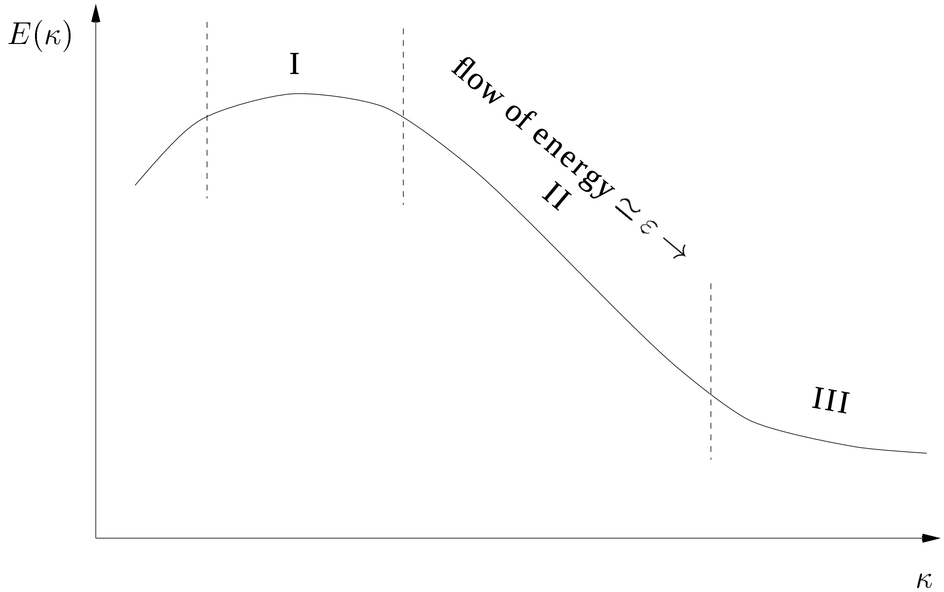


• The scales of the smallest eddies is determined by:

1) viscosity ν $[\mathrm{m}^2/\mathrm{s}]$
2) increase of internal energy = dissipation = ε [energy/time] = $[\mathrm{m}^2/\mathrm{s}^3]$

Thus the length scales η of the smallest eddies can be expressed as

$$\begin{array}{ccccc} \eta & = & \nu^a & \varepsilon^b \\ [m] & = & [m^2/s] & [m^2/s^3] \end{array} \tag{61}$$



We get two equations, one for meters $[m]$ and one for seconds $[s]$

$$
\begin{aligned}
m : 1 &= \;\; 2a + 2b \\
s : 0 &= -a - 3b,
\end{aligned}
$$

which gives $a = 3/4$ and $b = -1/4$, so that

$$\eta = \left(\frac{\nu^3}{\varepsilon}\right)^{1/4}.$$

The length scale η is called the *Kolmogorov* length scale. Writing the velocity scale and the time scale in the same way as Eq. 61, gives the Kolmogorov velocity scale υ and the Kolmogorov time scale τ as

$$\upsilon = (\nu\varepsilon)^{1/4}, \;\; \tau = \left(\frac{\nu}{\varepsilon}\right)^{1/2}$$

Turbulent Spectrum

The figure above shows the kinetic energy of turbulent eddies of different size. The wave number κ is inversely proportional to the wavelength λ which in turn is proportional to the length scale of the eddy; thus

$$\kappa \propto \frac{1}{l}.$$

Above we discussed the cascade process visualizing it in different size of eddies. Here, we discuss the cascade process in connection to the turbulent spectrum.

Large energy containing eddies are represented in Region I, see the figure above, which corresponds to small κ and large l. Kinetic energy is transported in spectral space (in wave-number space) from large scales (Region I: small κ, large l) to small scales (Region III: large κ, small l), via the intermediate scales (Region II). At the small scales (Kolmogorov scales η, υ and τ), the dissipation is proportional to the velocity gradient $(\partial u'/\partial x)^2$, where u' is the velocity scale of wave number κ. This velocity gradient increases as κ increases (will be shown in a later lecture). Thus, dissipation ε takes place at the small scales.

In the intermediate range (Region II), or the inertial region, the turbulent scales are independent both of the large scales and the small scales if the Reynolds number $Re_\ell = \mathcal{U}\ell/\nu$ is large. This region is characterized by the amount of energy being transported through the spectrum per time unit (i.e. ε) and the size of the eddy (i.e. $1/\kappa$). Thus the energy of an eddy in this region can be estimated as

$$E \propto \varepsilon^a \kappa^b$$

Dimensional analysis gives $a = 2/3$ and $b = -5/3$ so that

$$E = c\varepsilon^{2/3}\kappa^{-5/3}$$

The expression above is very famous, and it is called the *Kolmogorov specrum law* or $-5/3$ *power law*.

If we integrate the kinetic energy for wave number κ (i.e. $E(\kappa)$) over the whole wave space we get the turbulent kinetic energy k

$$k = \int_0^\infty E(\kappa) d\kappa.$$

k represents the integrated effect of all scales, but since the energy $E(\kappa_I)$ of the energy containing eddies (Region I) is the most important we have

$$k \simeq \int_0^{\kappa_I} E(\kappa) d\kappa.$$

The turbulent kinetic energy k is defined as (see Eq. 3.4 in V & M)

$$k = \frac{1}{2}\left(\overline{u^2} + \overline{v^2} + \overline{w^2}\right)$$

- Estimation of ε

If we assume that *all* kinetic energy is dissipated at small scales, it follows from the cascade process that the amount of energy extracted by the large scales from the mean flow is equal to ε. Hence, ε can be estimated by the large scales $\mathcal{U}$ and ℓ. A large eddy looses its energy to a slightly smaller one during the life time of the eddy t so that

$$\varepsilon = \frac{\mathcal{U}^2}{t}.$$

The time t must be related to the time scale of the large eddy $\ell/\mathcal{U}$ which gives

$$\varepsilon = \frac{\mathcal{U}^3}{\ell}.$$