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Highlights of Turbulence Modeling. . .

. . .Or, You Don’t Get What You Don’t Pay For

Peter Bradshaw

Mech. Engg. Dept., Stanford University
Stanford, CA 94305-3030
bradshaw@vk.stanford.edu

“COURSE” OUTLINE – I

• Bradshaw, Wilcox (FE-9) and Ferziger (FE-12) – a Package Deal

• Turbulence at advanced graduate level – not for turbulence specialists

• Most predictions of turbulence are made by non-specialists...

... i.e. design or development engineers for whom turbulence is only part of the problem

• The wrong prediction method can give **very** wrong predictions

"COURSE" OUTLINE – II

- This lecture is an introduction to the problem. . .and the possible solutions
- Dr. Wilcox will discuss Reynolds-averaged (time-averaged) modeling. No model can be trusted completely!
- Prof. Ferziger will discuss large-eddy simulation – the larger eddies are calculated exactly and only the smaller ones are modeled.
- LES is much more expensive than Reynolds-averaged modeling but may be the wave of the future

IT'S DIFFICULT – WHY BOTHER?

• Turbulence is the most complicated kind of fluid motion...

...but it is also the most common, on all scales from cream in coffee to the motion of the Galaxy...

...including most fluid flows found in mechanical engineering.

• The first 'bullet' implies that turbulence is the general solution of the Navier-Stokes (NS) equations – so why not just solve them?

CONTENTS

⇒ **Why turbulence is difficult – a little physics**

• Why engineers need short cuts – a little economics

• Short cuts:– large-eddy simulation, Reynolds-averaged modeling

• Hierarchy of Reynolds-averaged models:– eddy viscosity with zero, one, two...PDEs; stress-transport equations

• What of the future?

TURBULENCE IS A TANGLE

• Flow visualization of turbulence (smoke in air, dye in water) shows a wide range of length scales in all three axis directions.

• The largest motions ("eddies") nearly fill the flow... ...while the smallest may be too small for the visualization to resolve.

• Turbulence is a 3D tangle of elementary vortices which tend to stretch each other – the "drunkard's walk"

• The elementary vortex lines and sheets get thinner until viscous diffusion balances stretching on the average – i.e. **statistically**.

TURBULENT ENERGY

• Mean-flow kinetic energy is transferred to turbulence because mean-flow distortion (e.g. shear) does work against the turbulent stresses $-\rho \overline{u'_i u'_j}$

• The N-S equations lead to a conservation equation for turbulent kinetic energy per unit mass $k \equiv (\overline{u'^2} + \overline{v'^2} + \overline{w'^2})/2 \equiv \overline{{u'}_i^2}/2$

• Vortex stretching "cascades" TKE to the smallest eddies where viscous stresses dissipate it into thermal internal energy.

• The mean flow transports TKE in space – and so does the turbulence itself ("diffusion")

TYPICAL SCALES – I

• We can produce *exact* definitions of *order-of-magnitude* scales

• The smallest wavelengths are of order $\eta \equiv (\nu^3/\epsilon)^{1/4}$, where ϵ is the rate of dissipation of TKE per unit mass. Close to a solid wall η is about 1.5 “wall units”

○ The large eddies determine the dissipation rate ϵ: the smallest eddies just do the dissipating

• The largest wavelengths are of the order of the flow width (they determine it!). $k^{3/2}/\epsilon$ is a representative scale – about 0.6δ in the outer part of a boundary layer.

TYPICAL SCALES – II

• The *friction velocity* $u_\tau \equiv \sqrt{(\tau_w/\rho)} \equiv u_e\sqrt{(c_f/2)}$ and the thickness δ are useful global scales for wall flows

• $\sqrt{(\tau/\rho)}$ is a pointwise velocity scale of the larger eddies. . .

. . .but $\sqrt{k}$ is more useful velocity scale. It also varies across the flow. . .

. . . in the inner part of a boundary layer $\sqrt{k}$ is typically about twice the friction velocity

HEAT TRANSFER

• Many engineers are concerned with heat transfer rather than momentum transfer. . .

. . .but of course the velocity field must be calculated concurrently

• We can construct a heat-transfer **analogy** of any turbulence (momentum-transfer) model

• The simplest is (*eddy conductivity*) = (*eddy viscosity*)/Pr_t, with a formula for turbulent Prandtl number Pr_t

• Most advances in modeling appear first in momentum transfer and are then adapted to heat transfer

COMPRESSIBLE FLOW

• Incompressible models do well in shock-free attached flows (you need a coupled heat-transfer calculation for the density)...

...strictly, they do as well as the data

• Shock interactions (or even strong pressure gradients) defeat most models – heat transfer predictions are bad

• Turbulence structure of mixing layers changes greatly with Mach number (not density ratio) and M-dependent model coefficients are needed

TRANSITION – I

• Many engineers are concerned with low-Re turbulent flows...
...which are greatly influenced by the initial transition from laminar to turbulent flow

• This depends on the background disturbance field, which is almost never completely known in practice

• Surprisingly, erratic transition is not too important in engineering...

...partly because background disturbances are often so large that transition occurs quickly

• Most engineers use empirical correlations

TRANSITION – II

• Reynolds-averaged turbulence models cannot be expected to predict instability to *small* disturbances

• "Bypass" or "breakthrough" transition with large background turbulence is a more hopeful case...

...but the structure of a transitional flow is not much like a fully-turbulent flow

• Wilcox is more optimistic than I am – I would stick with empirical correlation formulas for now

GOOD NEWS / BAD NEWS

• If the viscosity is small (high Reynolds number) the smallest motions are *very* small (though still much bigger than the molecular mean free path, so the NS equations are valid)

• Bad news – if a finite-difference solution of the NS equations, in x, y, z, and t, is to resolve the smallest eddies...

...grid size must be orders of magnitude less than flow width.

• Things get worse as the Reynolds number increases – ratio of flow width to smallest-eddy scale (η) is proportional to $Re^{3/4}$

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⇒ **Why engineers need short cuts – a little economics**

• Short cuts:– large-eddy simulation, Reynolds-averaged modeling

• Hierarchy of Reynolds-averaged models:– eddy viscosity with zero, one, two...PDEs; stress-transport equations

• What of the future?

DIRECT NUMERICAL SIMULATION – I

• Solution of the NS equations in x, y, z, and t for the whole range of eddy sizes is called **Direct Numerical Simulation** (DNS).

• Computing time and cost restrict DNS to low Reynolds numbers (cost $\propto$ Re^3 approx., where 3 = 3/4 $\times$ **4**)

• The affordable Re rises (slowly) as computing gets cheaper...

...**today** the limit corresponds to simple flows at small scales (DNS is starting to replace lab. experiments)...

DIRECT NUMERICAL SIMULATION – II

• Computing requirements for DNS at the Reynolds numbers of aircraft or ships are stupefyingly large

• In the absence of an improvement in computer power by factors of 10^{12} or so, DNS is not an option for such cases

• Today DNS is useful in principle for low-Re flows in engineering. . .

. . .but has the same difficulties as approximate methods in predicting transition

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SHORT CUTS: MODELING

• "Modeling" of turbulence means replacing unknown terms in some exact equation(s) by empirical formulas (or equations) calibrated with data from experiments (or DNS)

• The object is to reduce the number of unknowns to equal the number of equations – i.e. "close" the system of equations

• Completely- ("Reynolds")-averaged models are often classified by the number of *turbulence* equations

• Usually we just want global averaged quantities – skin friction, pressure drop, boundary-layer thickness and heat transfer

SHORT CUTS: LARGE EDDY SIMULATION – I

• Most of the turbulent transfer (mixing) of mass, momentum and heat is carried out by the larger eddies. . .

. . .the smallest eddies just dissipate TKE into heat

• Computing cost can be greatly reduced by calculating large eddies, as in DNS, but modeling small ones – this is LES.

• Models for the small ("sub-grid-scale") eddies are usually quite simple – if the small eddies bear little Reynolds stress, all the SGS model has to do is dissipate energy

SHORT CUTS: LARGE EDDY SIMULATION – II

• Near a solid surface ALL eddies are small, so either:–
(i) LES in this region becomes as expensive as DNS, or
(ii) the SGS model must do the whole calculation, or
(iii) a "boundary" condition must be used to terminate the LES at some distance from the surface, or
(iv) (current practice) compromise between (i) and (ii) – use a grid which is somewhat coarser than needed for DNS (but Re-dependent in the viscous wall region)...
...and use a moderately sophisticated SGS model, possibly a self-calibrating "dynamic" model

SHORT CUTS: REYNOLDS-AVERAGED MODELS

• Osborne Reynolds decomposed variables in turbulence into a *mean* (e.g. a time average) plus a *fluctuation*, e.g. $u + u'$

• Taking the mean of the NS equations leaves the mean rates of transfer of momentum by the turbulence as extra unknowns in the "RANS" equations. Move them to the r.h.s. of the equations and call them gradients of the (Reynolds) **stresses**

• The apparent stress $-\rho\overline{u'v'}$ acts in the x direction (for u') on a plane normal to the y direction (for v'), and so on for all $u'_i u'_j$

• Current engineering calculation methods are mostly Reynolds-stress models (not called "simulations")

TRANSPORT EQUATIONS

• Turbulence quantities like Reynolds stresses obey exact PDE "transport equations" derivable from the NS equations...

...but with further unknowns that must be *modeled*

• $D\overline{u'_i u'_j}/Dt$ has dimensions [*velocity*^{2}/*time*] or [*velocity*^{3}/*length*]...

...so we need a model transport (or other) equation to give a length scale or time scale. Common variables are ϵ or ϵ/k

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HIERARCHY OF REYNOLDS-AVERAGED MODELS

• Many engineering flows are dominated by boundary layers or other "thin" shear flows...

...dominated by the shear stress in the plane of the mean shear

• Turbulent stresses are related to the **mean-flow history**, but large mean shear usually implies large local shear stress...

...and quite simple empirical formulas relating turbulent shear stress $-\rho\overline{u'v'}$ to mean shear $\partial u/\partial y$ give adequate predictions

• Alas the empirical coefficients have to be changed from one flow to another, by large amounts in complex flows.

EDDY VISCOSITY

• "Eddy viscosity" is **defined** as the ratio of a Reynolds stress to the mean rate of strain in the same plane. It is *measurable*

• Simplest example: $-\overline{u'v'} = \nu_t \partial u/\partial y$

• The full eddy-viscosity formula for constant-density flow is

$$-\overline{u'_i u'_j} + \delta_{ij}\overline{u'^2_i}/3 = \nu_t(\partial u_i/\partial x_j + \partial u_j/\partial x_i)$$

where $\delta_{ij} = 1$ if $i = j$ and zero otherwise

• Alas ν_t varies from place to place and from flow to flow and is different for different stresses... sometimes it is negative!

EDDY VISCOSITY – NOT AS SILLY AS IT SEEMS

• Eddy viscosity is useful **if** it varies more predictably than the Reynolds stresses

• Reynolds stresses are generally large where mean velocity gradients are large...

(e.g. we said $-\overline{u'v'}$ tends to be large where $\partial u / \partial y$ is large)

• That is, eddy viscosity usually varies less rapidly than the stresses – which *may* mean "more predictably"

• Consider a dog on a long leash – it follows roughly the same path as its master, but can get up to mischief on the way

EDDY VISCOSITY – A MONGREL DOG

• Eddy viscosity is the ratio of a turbulence quantity (Reynolds stress) to a mean-flow quantity (mean rate of strain)

• Correlating it entirely in terms of mean-flow scales or entirely in terms of turbulence scales is justifiable only if the two are proportional...

...a working definition of a "equilibrium" flow

• Nevertheless, most turbulence models in commercial use are based on an eddy viscosity, assumed to be a *scalar* (same for all stresses)

“ZERO-EQUATION” MODELS

... a.k.a. “Algebraic Eddy Viscosity” models

• Good results are obtained in simple cases by taking $\nu_t \propto y$ near a solid surface, so that the logarithmic “law of the wall” is reproduced...

...while in the outer part of simple boundary layers the empirical relation $\nu_t = 0.0168 u_e \delta^*$ (or equivalent) works well even in non-equilibrium cases (e.g. strong pressure gradient)

• In more complex flows, mean-flow length scales like δ^* may not be definable, the log law may not work...

...and most general-purpose models go to the other extreme, parameterizing eddy viscosity with turbulence scales

EDDY-VISCOSITY-TRANSPORT MODELS

• We can **define** a coefficient c_μ by $\nu_t = c_\mu k^2/\epsilon$

where k and ϵ are TKE and dissipation rate ("typical scales")

• Then modeling and solving the transport equations for k and ϵ gives a "two-equation" model for ν_t

• The catch is to determine c_μ! Remember the mongrel dog

THE LOGARITHMIC LAW

• Recall that taking $\nu_t \propto y$ near a solid surface yields the logarithmic law...

...and u/u_e is as much as 0.7 at the outer limit of the log. law

• Forcing an eddy-viscosity-transport model to reproduce $\nu_t \propto y$ is a powerful constraint on the empirical coefficients, including c_μ...

...and all the better models have this constraint

• Some people might call it cheating!

TYPICAL SCALES – III

• The typical velocity scale $k^{1/2}$ and length scale $k^{3/2}/\epsilon$ (or time scale k/ϵ)...

...can be used in modeling (recall the definition of c_μ). (These are most popular scales, not the only ones in use)

• Example – the ϵ transport equation itself contains a viscous destruction term, with dimensions of [ϵ/time]. We model this as $c_{\epsilon_2}\epsilon^2/k$, where c_{ϵ_2} is a dimensionless empirical coefficient

• Example – the equation for $k \equiv \overline{u_i'^2}/2$ has a turbulent transport ("diffusion") term $-\partial\overline{u_i'^2 u_l'}/\partial x_l$ and $\overline{u_i'^2 u_l'}$ is usually modeled by "gradient transport" $-\nu_k \partial\overline{u_i'^2}/\partial x_l$...

... where $\nu_k = \nu_t/\sigma_k = c_\mu(k^2/\epsilon)/\sigma_k$

THE $k^m \epsilon^n$ FAMILY – I

• Several models use a second variable other than ϵ, but all are of the form $k^m \epsilon^n$... (we need ϵ explicitly in the k equation)

• If we transform a model $k^m \epsilon^n$ equation to one for ϵ, say, using the model k equation...

...the ϵ equation is different for different m and n – in the form of the terms and not merely in the coefficients. There must be an optimum m and n, in some sense

• The exact transport equations for ϵ or $k^m \epsilon^n$ inspire only fear – so the model equations are almost entirely empirical

• The most popular "two-equation" model – probably not the best – is the most straightforward, using k and ϵ

THE $k^m \epsilon^n$ FAMILY – II

• A few models use fractional powers of k as the *first* variable – a k equation can be recovered but has extra terms not found in the exact k equation, and this is unappealing

• Choices for second variable include Wilcox's ω (a constant times ϵ/k) and its inverse, the time scale k/ϵ.

• Information about the effect of a solid wall on eddy length scale propagates away from the wall...

...so if turbulent transport of $k^m \epsilon^n$ is to be modeled by gradient diffusion, $k^m \epsilon^n$ should decrease with increasing y – in practice, n should be positive

• ϵ/k ($m = -1$, $n = 1$) seems near optimum in several respects

MORE – OR LESS – EQUATIONS BETTER?

• The model transport equations for (e.g.) k and ϵ could be combined to give one equation for k^2/ϵ and hence ν_t

• There are several wholly-empirical transport equations for ν_t

• The Spalart-Allmaras ν_t model seems competitive with the best two-equation models and is in principle cheaper to solve

• Even two equations is not many to describe turbulence! Durbin's "$v2f$" model uses a further variable, equal to the normal stress $\overline{v'^2}$ near a solid surface, in addition to k and ϵ...

...and also an empirical *elliptic* equation to represent the effect of pressure fluctuations (which obey a Poisson equation)

NONLINEAR EDDY-VISCOSITY FORMULAS –I

• Several nonlinear formulas have been suggested, including products of the strain rate $S_{ij} = (\partial u_i/\partial x_j + \partial u_j/\partial x_i)/2$ and the "rotation tensor" $\Omega_{ij} = (\partial u_i/\partial x_j - \partial u_j/\partial x_i)/2$ up to third order

• These formulas are subject to invariance constraints and symmetry requirements but still include many extra adjustable terms

• Obviously the eddy viscosity may behave badly if S_{ij} and/or Ω_{ij} becomes large, so products become very large – this worries me!

NONLINEAR EDDY-VISCOSITY FORMULAS – II

• Six empirical coefficients adequately define my clothes (collar, arm, chest, waist, leg, foot)...

...but turbulence is more complicated (a warning to those who buy turbulence models off the peg?)

• The extra coefficients in nonlinear eddy-viscosity models may improve agreement with experiment even if the extra terms do not correspond to specific phenomena (e.g. streamline curvature)...

...which worries me even more!

• Eddy viscosity is not a reliable enough concept to carry "bells and whistles" like this

ALGEBRAIC STRESS MODELS – I

• These yield nonlinear, anisotropic eddy viscosity but are stress-transport models with simplified transport terms.

• The usual simplification is to assume that the mean-transport and turbulent-transport terms in each stress-transport PDE are proportional to the stress being transported...

...it follows that the transport terms need be modeled for only one stress equation – always the TKE equation...

...and the others reduce to (simultaneous) algebraic equations for the stresses

• A big publicity point for ASM is the ability to predict stress-induced secondary flows in non-circular ducts, at least qualitatively

ALGEBRAIC STRESS MODELS – II

• Unfortunately, the exact stress-transport equations show that the assumption that the mean-transport terms are proportional to the stress being transported is poor in rapidly-changing flows

• Here, large mean transport of $-\overline{u_i'u_j'}$ comes from large generation $(\overline{u_i'u_l'}\partial u_j/\partial x_l + \overline{u_j'u_l'}\partial u_i/\partial x_l)\ldots$

...so for a 2D shear layer the generation terms proportional to $\overline{u'v'}$ are $(\overline{u'v'}\partial v/\partial y + \overline{u'v'}\partial u/\partial x)$ which sum to zero by continuity!

• The complete turbulent-transport term is not closely proportional to the transported stress (not its *gradient*), especially near free-stream edges where turbulent transport is largest.

STRESS-TRANSPORT MODELS – I

• The transport equations for the Reynolds stresses (four nonzero in 2D flow, six in 3D) can be modeled directly...

... with a transport equation for ϵ or other quantity giving a length scale

• In the late 1960s stress-transport (sometimes called "second moment") modeling seemed to be the wave of the future – solve equations directly for the quantities you want

• However, stress-transport models have proved disappointing – they often do better than isotropic eddy-viscosity models, but do not deliver engineering accuracy over a wide range of flows...

STRESS-TRANSPORT MODELS – II

• Most models use an ϵ equation, essentially the same as in two-equation models, to provide a length scale...bad news?

• Parneix and Durbin (1996) showed that the k, ϵ dissipation equation does “surprisingly well” in a backstep flow

• Experience with the equation for $\omega \equiv \epsilon/k$ is mixed

• It is difficult to defend using different length-scale equations in 2-equation and stress-transport models...

...but disbelievers in eddy-viscosity models are entitled to reject their (almost-entirely-empirical) length-scale equations and start again!

STRESS-TRANSPORT MODELS – III

• A major problem is modeling the "pressure-strain" (redistribution) term $\overline{p'(\partial u'_i/\partial x_j + \partial u'_j/\partial x_i)}$ in each $\overline{u'_i u'_j}$ equation (zero in the summed k equation)

• The pressure-strain term cannot be reliably measured: DNS results are some help

• Most models parameterize it as a function of local quantities, but p' is an integral of a Poisson PDE over the whole flow (including the "image" flow beneath a solid surface)

• Durbin's semi-empirical elliptic PDE for the pressure-strain term is based on the Poisson equation – a qualitative improvement

STRESS-TRANSPORT MODELS – IV

• Stress-transport models supply the Reynolds stresses to the mean-flow transport equations as new source terms...

...whereas eddy-viscosity models supply a factor in existing terms involving the mean rates of strain

• Thus the mean-flow and turbulence equations are less closely coupled in stress-transport models, and therefore tend to be "stiff"

• More steps, more equations, more money

• Also, eddy-viscosity codes need major revision to incorporate stress-transport models (because the stresses are supplied differently), so users of the former are reluctant to change to the latter

“LOW-RE” MODELS...

• Near a solid surface, flow structure and optimum model coefficients depend on $u_\tau y/\nu$ – a dimensionless distance from the surface *and* a Reynolds number

• Influence of solid surface is partly viscous (from $u, w = 0$), partly inviscid “blockage” (from $v = 0$)

• Either way, use $u_\tau y/\nu$ or $k^2/(\epsilon\nu)$ to correlate

• Durbin’s elliptic equation for the pressure-strain term expresses the blockage effect...

... and, given $u = v = w = 0$ at $y = 0$, it seems to avoid the need for “damping functions” (so does k, ω)

...AND WALL FUNCTIONS

• "Integration to the wall" is expensive (small y steps, greater stiffness) – so as usual we want something cheaper

• In 2D simple flows, $u/u_\tau = f(u_\tau y/\nu)$ for $y/\delta < 0.1 - 0.2$ and $\overline{u'^2}/u_\tau^2$, etc., is constant

• Apply these "law-of-the-wall" relations as off-the-wall boundary conditions

• Catch is that in 3D or non-simple flows these relations can be highly inaccurate – and the numerics can be a pain

• Unfortunately nothing guarantees that a "low-Re" model calibrated to reproduce the law of the wall is accurate where the latter isn't

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⇒ **What of the future?**

WILL REYNOLDS-AVERAGED MODELING DELIVER?

• No current model can be relied on to produce results of engineering accuracy over the full range of flows

• Usually, the results are more accurate than an expert's guess and are therefore valuable guides for designers. . .

. . .but computers are nowhere near replacing wind tunnels as promised 25 years ago

• Reynolds averaging is a brutal simplification – s little hope of a model that gives engineering accuracy for all flows?

• Optimizing a model to give good results in the sort of flows that interest you today is fine – *today!*

IMPROVEMENTS IN REYNOLDS-AVERAGED MODELING

• More-elaborate eddy-viscosity relations have even less physical foundation than linear eddy viscosity...

...and are surely accidents waiting to happen

• Allowances for flow history are more important...

...and stress-transport models *ought* to do better than eddy-viscosity models

• It may be worth asking *"Why not?"*

LES TO THE RESCUE?

• LES can already be relied on to predict free turbulent flows to good engineering accuracy, at any Reynolds number. . .

. . .but the viscous wall region demands *either* an off-the-wall boundary condition *or* a sub-grid-scale model carrying most of the Reynolds stress, and both involve empirical approximations.

• The only alternative is to refine the near-wall grid so that the LES becomes a DNS – with severe Reynolds-number limitations

• This is undoubtedly the “pacing item” in LES – and possibly in turbulence modeling in general

LES/RANS HYBRIDS?

• Many people agree that expensive Large-Eddy Simulation will be used only in difficult parts of the flow. . .

. . .with the "easy" parts left to Reynolds-averaged models

• Big problem is providing fluctuating inlet data for LES from Reynolds-averaged statistics alone. . .

. . .perhaps by starting LES upstream of where it is really needed and rescaling after it has settled down

CONCLUSIONS – I

• Most current Reynolds-averaged models are based on PDE "transport" equations for isotropic (scalar) eddy viscosity…

…but eddy viscosity in real flows is not isotropic and often behaves erratically

• Model transport equations for the Reynolds stresses themselves are more realistic in principle…

…but sometimes behave erratically when they shouldn't

CONCLUSIONS – II

• Wilcox will talk about what works and what doesn't

• I have more hope than he has that the reliability of stress-transport models can be improved...

...but I fear that Reynolds-averaged models will only improve slowly, eventually becoming junior partner to LES

CONCLUSIONS – III

• Turbulence modeling is important to many mechanical engineers

• Current turbulence models are better than nothing. . .

. . .but not as reliable as design tools should be

• The user of a turbulence model is more like a test pilot than a sunny Sunday Cessna flier

TURBULENCE BIBLIOGRAPHY

- Bradshaw's Web page at

http://vonkarman.stanford.edu/tsd/resp_b.html

leads to a bibliography of turbulence and related subjects made up of annual files, totaling about 10000 entries (2MB), each with a one-line abstract, plus the same information sorted into index categories

- Files are updated monthly

WEB VERSION

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also leads to the .html version of these notes, with references to reviews of specialized subjects