# PID Controller Design for Fractional-Order Systems with Time Delays

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**Abstract**

Classical $\mathcal{H}_\infty$-stabilizing PID controllers are designed for linear time invariant fractional systems and the effect of input-output time delay on the range of allowable controller parameters is investigated. The allowable PID controller parameters are determined from a small gain type of argument used earlier for finite dimensional plants.

## 1 Introduction

The analysis and control synthesis of fractional order systems have received an important attention in the last years due to its importance among various engineering applications. Many design procedures have been developed to deal with the design of fractional order controllers for both rational (finite dimensional) and fractional systems, e.g. [1, 2] just to be concise. Analogously, there exists many different PID controller design techniques for rational (finite dimensional) systems with delays, for example [3, 4, 5].
The main objective of this work is the design of *classical* PID controllers by extending the approach of [3] to fractional order systems with time delays.

## 2 Main Results

We consider systems represented by transfer functions of the form

$$P(s) = e^{-hs}\frac{G(s^\alpha)}{s^\alpha - p} \tag{1}$$

where $h > 0$ is the input-output time delay, $\alpha \in (0, 1)$ is the fractional order, $p \geq 0$ is the location of the unstable pole of the plant and $G(\omega)$ is a rational stable transfer function in the variable $\omega = s^\alpha$ with $G(p) \neq 0$ and $G(0) \neq 0$.
We consider the standart single input single output scheme, and our goal is to design a classical Proportional + Integral + Differential (PID) controller in the form

$$C(s) = K_p + \frac{K_i}{s} + K_d\frac{s}{\tau_d s + 1} \tag{2}$$

where $K_p$, $K_i$ and $K_d$ are free parameters and $\tau_d$ is an arbitrarily small positive real number making the controller proper.
As in [3], the design will be done in two steps, where first PD controllers are investigated and then the integral action is added. For their calculation, it will be used some arguments based on the small-gain principle.
The PID controller design method leads to optimization problems when we aim to maximize the range of the allowable parameters. Some of them can be solved by a one-dimensional numerical search.
The method is illustrated by means of a fully worked out example for the case where $\alpha = 0.5$ and $G(s) = 1$.

## References


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