On the stability of neutral linear systems with multiple commensurated delays

C. Bonnet*, A. R. Fioravanti*
INRIA Rocquencourt.

J. R. Partington†
School of Mathematics, University of Leeds.

Abstract

This work addresses the H_∞ stability analysis of neutral time-delay linear systems with multiple commensurate delays, including those with chain of poles asymptotic to the imaginary axis. Some frequency-domain tests for this kind of stability are proposed.

Keywords

Delay effects, Neutral Systems, H_∞ stability.

1 Introduction

In the frequency-domain approach to the stability of delay systems, it is the systems of neutral type that are the most difficult to analyze, since they may have chains of poles asymptotic to the imaginary axis in the complex plane. In this case, it is known [3] that all poles in the open left plane is a necessary but not sufficient condition for H_∞ stability. On the other hand, in the time domain, very recently, strong stabilizability of neutral delay systems was analyzed in [4].

We look at time-delay systems with transfer functions of the form

$$G(s) = \frac{t(s)}{p(s) + \displaystyle\sum_{k=1}^{N} q_k(s) e^{-ksh}}, \tag{1}$$

where $h > 0$, and t, p and q_k, for all $k \in \mathbb{N}_N = \{1, \ldots, N\}$, are real polynomials which satisfy $\deg p \geq \deg q_k$ and such that $\deg p = \deg q_k$ for at least one $k \in \mathbb{N}_N$. In order

*Domaine de Voluceau, B.P. 105 78153 Le Chesnay cedex, France. e-mail: {`Catherine.Bonnet, Andre.Fioravanti`}`@inria.fr`

†Leeds LS2 9JT, UK. e-mail: `J.R.Partington@leeds.ac.uk`

for (1) to be a proper neutral type delay system we assume also that $\deg p \geq \deg t$. According to the classification of delay systems found in [1], the pole chains of G have the following possible form:

1. If $\deg p = \deg q_N$, then there are only chains of neutral type;
2. If $\deg p = \deg q_k > \deg q_N$, for some $k \in \mathbb{N}_N$, then there are chains of both neutral and retarded types.

We extend some of the analysis of [3], which applies to denominators with a single delay term, basically the case $N = 1$ above.

2 Main Results

The stability of a system of type (1) is linked to the location of its poles. Those poles of small modulus can be found by numerical techniques (see, for example [2]), whereas for those of large modulus, the asymptotic behavior is crucial.

We start with some general analysis of the asymptotic behavior of the neutral chains of poles of linear time-delay systems with multiple commensurate delays. This is accomplished by finding analytical expressions for the location of poles with high norm, and consequently, the position of those poles in relation to the imaginary axis. With this information in hand, the next step is to characterize the H_∞ stability of this kind of systems, establishing in this way, easy to check conditions that guarantees this property.

We give one application of the theory developed here, which is to determine stability for delay-differential systems specified in matrix form. We also provide some computational results to illustrate and demonstrate the applicability of the proposed procedure.

One will notice that the essence of the analysis of poles of large modulus is somehow independent of the numerical value of the delay. Indeed, for such poles, only the relations between the delays are important. But the study of the remaining poles of small modulus keeps its delay-dependence nature.

References

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