Meeting 11

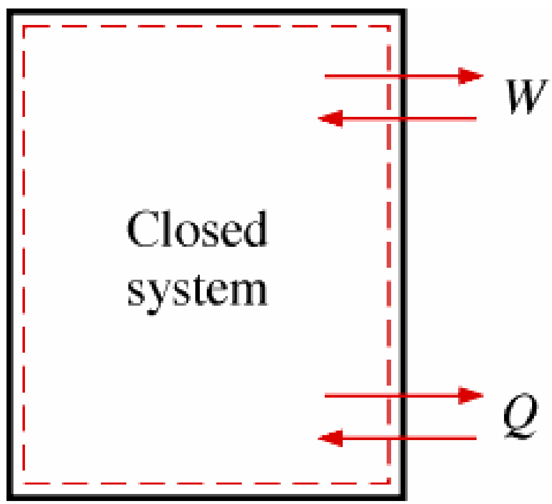
Chapter 5

Sections 5.1 & 5.2

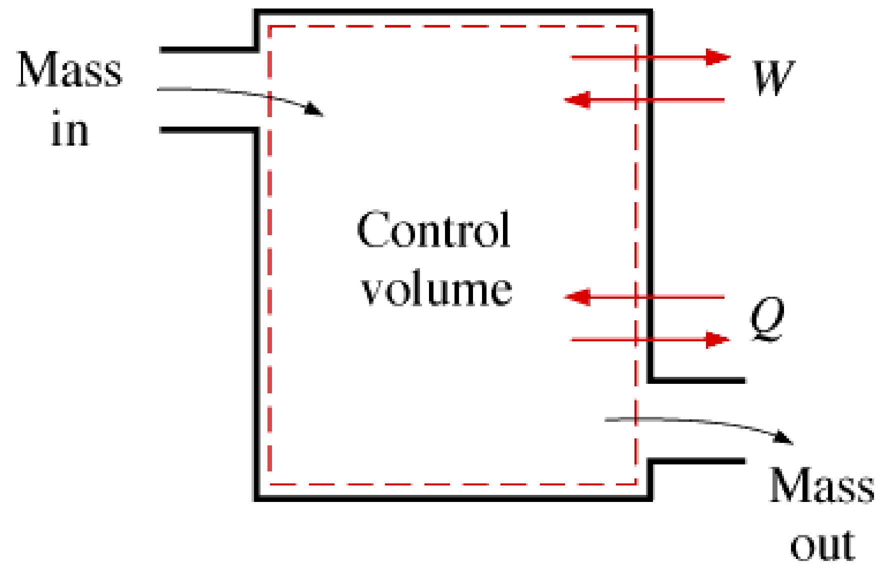
Open Systems Conservation Equations

- We now want to develop the conservation equations for an open system.
- What happens when the system is no longer closed, but something is flowing in and out of it?
- Need to determine how this will change our analysis from that of a closed system

Difference Between Closed and Open Systems



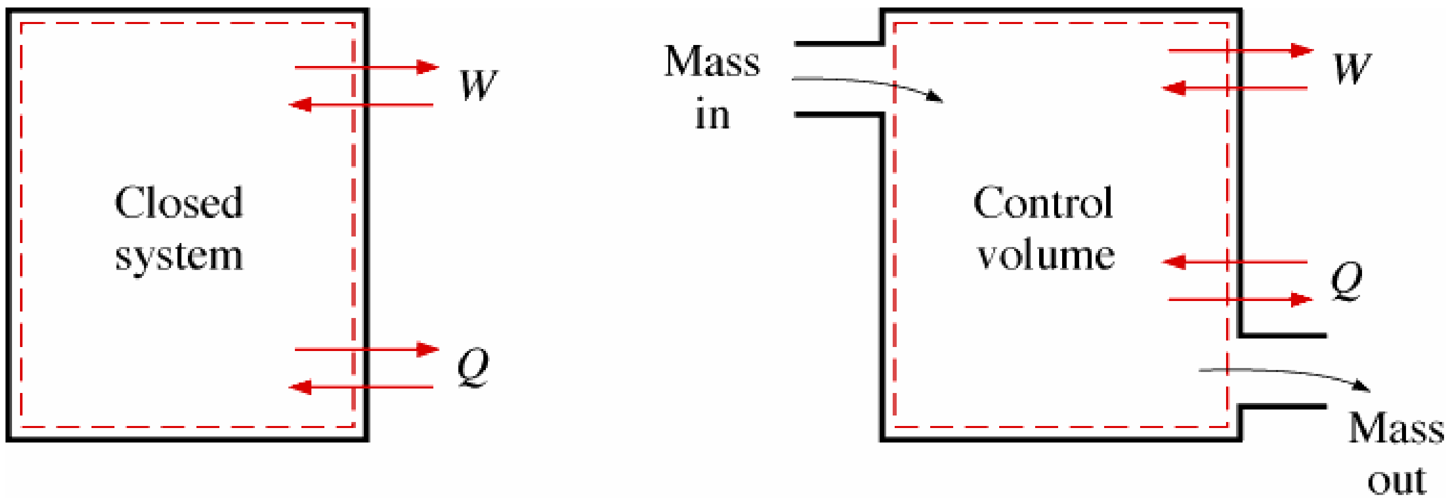
SYSTEM



CONTROL VOLUME

Mass Flow, Heat, and Work Affect Energy Content

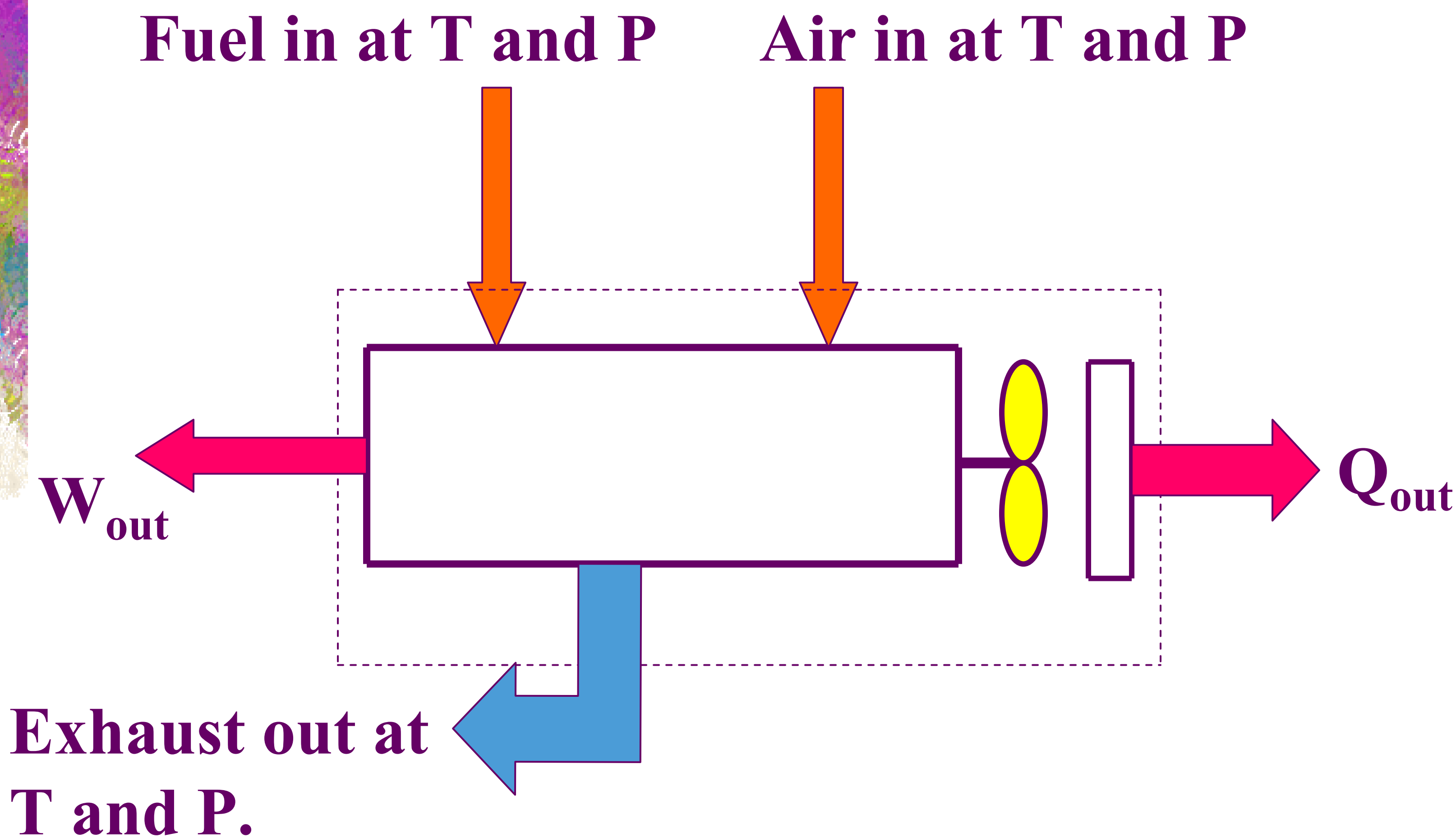
The energy content of a control volume can be changed by mass flow as well as heat and work interactions



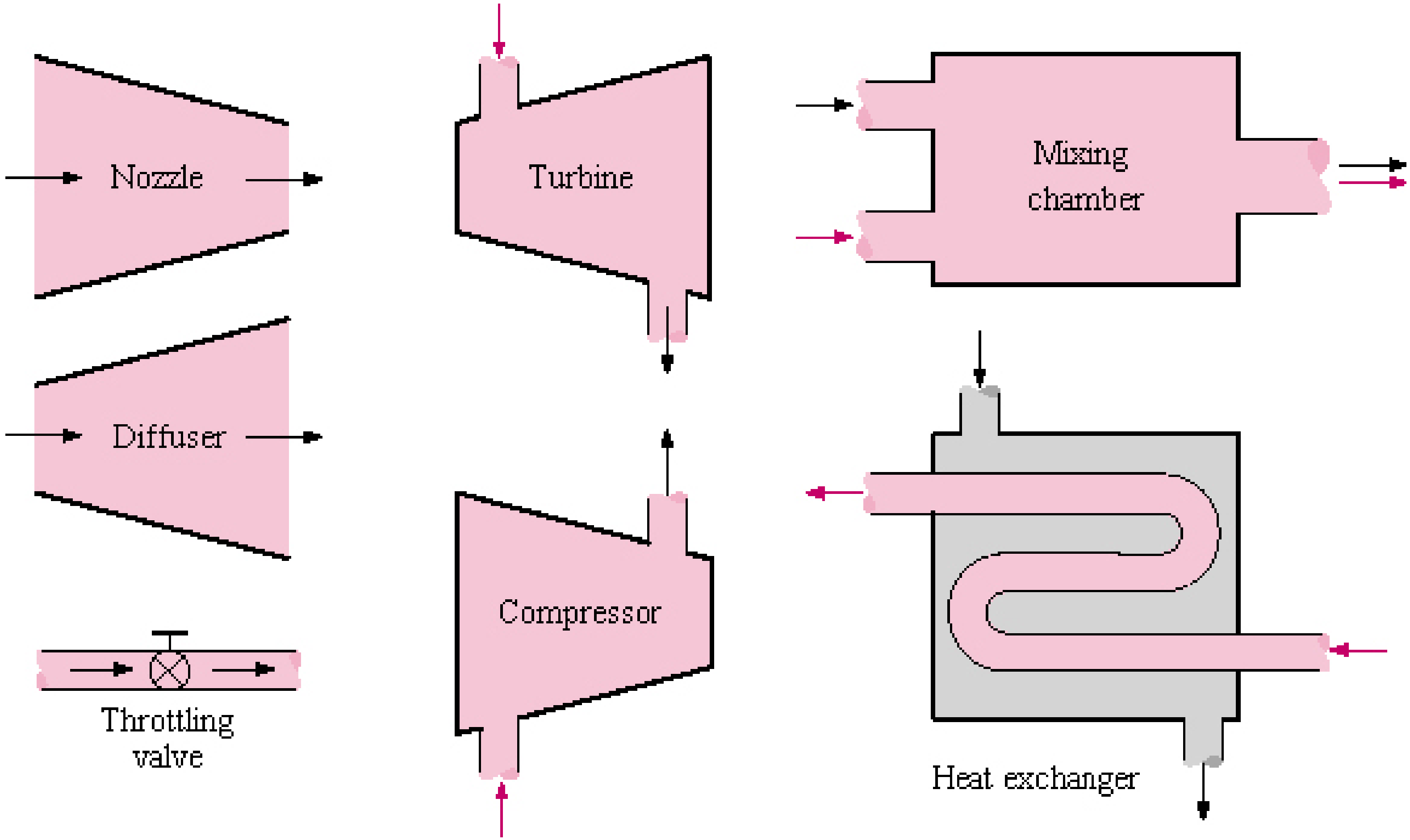
Control Volume

- System - control mass
- Control volume, involves mass flow in and out of a system
- pump, turbine, air conditioner, car radiator, water heater, garden hose
- In general, any arbitrary region in space can be selected as control volume.
- A proper choice of control volume will greatly simplify the problem.

Example – Automobile Engine



Control Volume



The Physical Laws and the System Concept

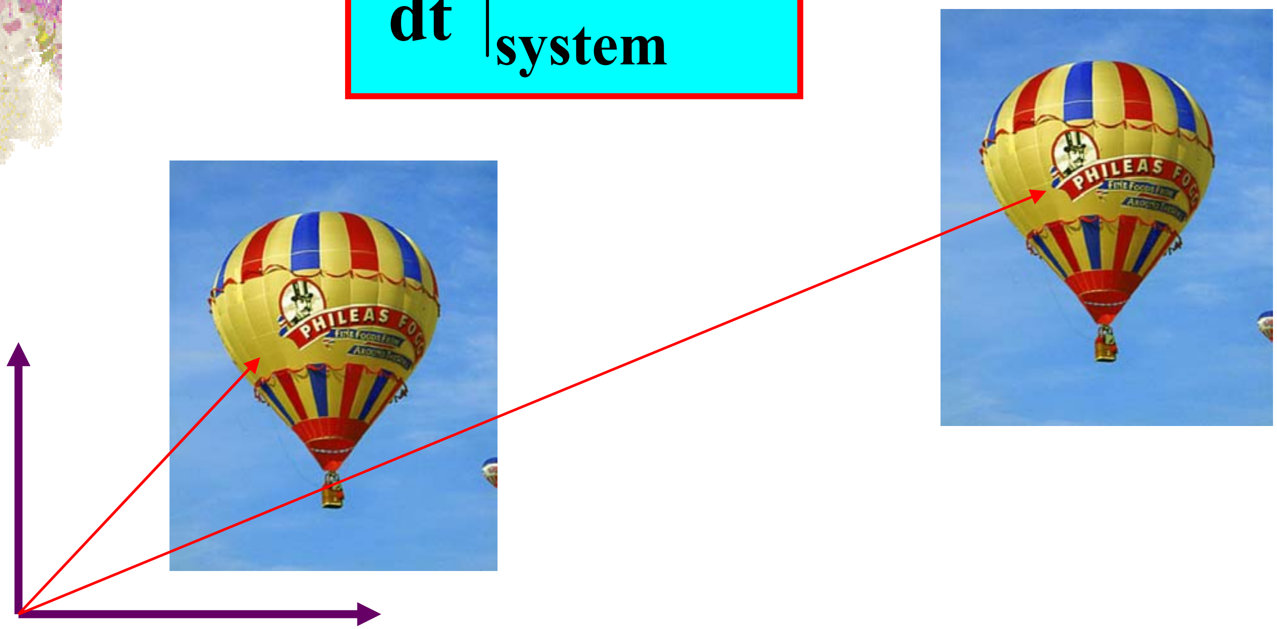
- **All physical laws seen so far were developed to systems only: a set of particles with fixed identity.**

- **In a system mass is not allowed to cross the boundary, but heat and work are.**

Mass Conservation Equation

- The mass within the system is constant. If you follow the system, in a Lagrangian frame of reference, it is not observed any change in the mass.

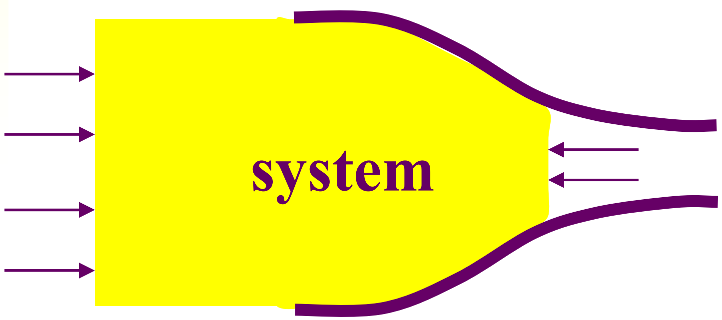
$$\left.\frac{dM}{dt}\right|_{system} = 0$$

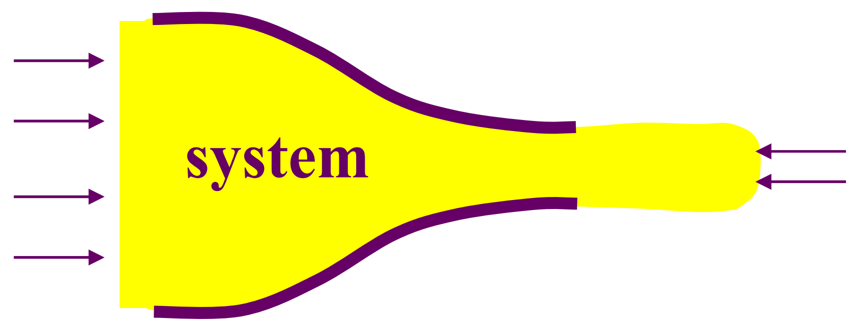


Momentum Conservation Equation

- **If you follow the system, in a Lagrangian frame of reference, the momentum change is equal to the resultant force of all forces acting on the system: pressure, gravity, stress etc.**

$$\left.\frac{d\left(M\vec{V}\right)}{dt}\right|_{system} = \underbrace{\sum \vec{F}}_{\substack{\text{external}\\ \text{forces}}}$$



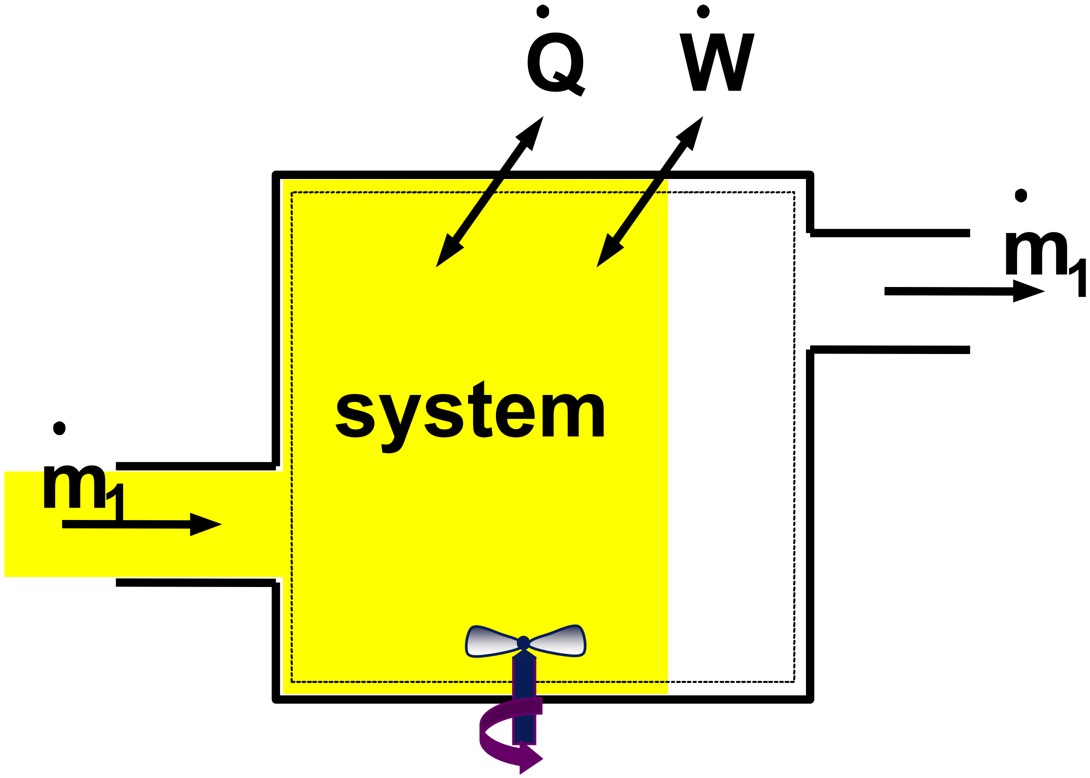


Energy Conservation Equation – 1st Law

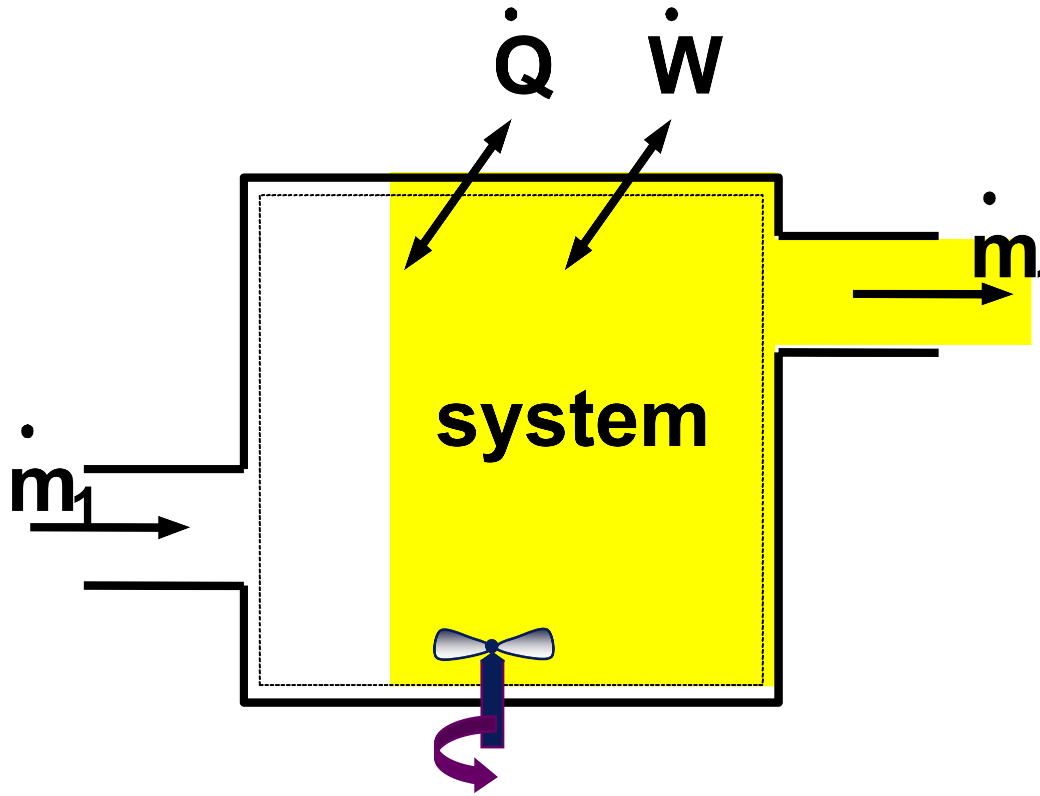
- If you follow the system, in a Lagrangian frame of reference, the energy change is equal to the net flux of heat and work which crossed the system boundary

$$\left.\frac{d(Me)}{dt}\right|_{\text{system}} = \oint\!\!\oint_{\text{boundary}} (\dot{q} - \dot{w})\,dA$$

- $e = u + gz + v^2/2$ specific energy (J/kg)
- $\dot{q}$ and $\dot{w}$ = energy flux, ($Js^{-1}m^{-2}$)



Instant: t_0



Instant: $t_0+\Delta t$

General Form of the Conservation/Transport Equations

$$\left.\frac{d(M\beta)}{dt}\right|_{system} = \text{Source Terms}$$

	B	β	Source
Mass	M	1	0
Momentum	MV	V	F_{ext}
1st Law	E	e	$(\delta q - \delta w)$

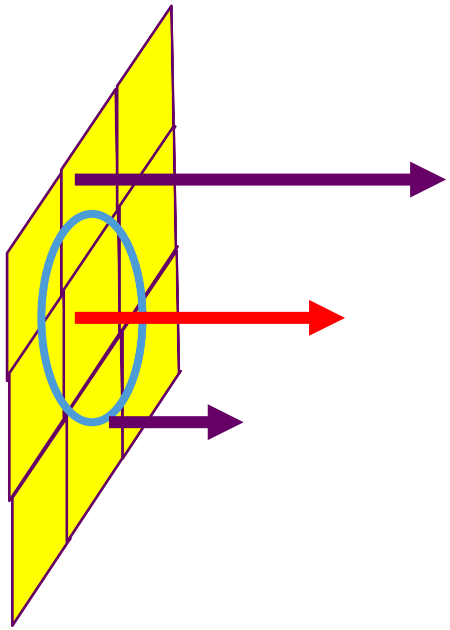
Systems x Control Volumes

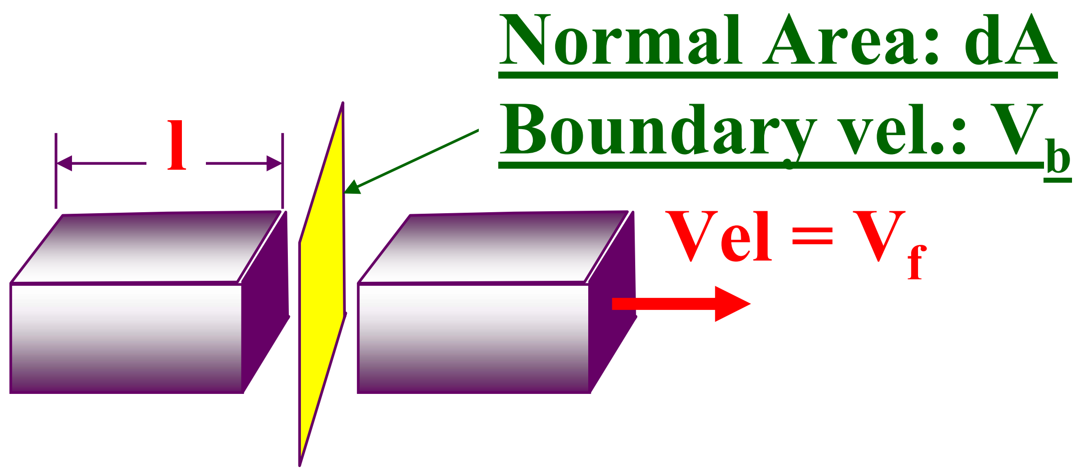
- **For continuously deforming boundaries (gases and liquids in general) is difficult to draw an analysis following the system.**
- **It would be far easier to have a fixed region in space (the control volume) and then draw the analysis.**

- ***How to transpose the system properties to the control volume properties?***

Preliminaries

- **Before get into the *Control Volume analysis* it is necessary define the mass flow in terms of the velocity.**





Time = t
Length = l
Area = dA
Fluid vel.: Vf
Boundary vel.: V_b

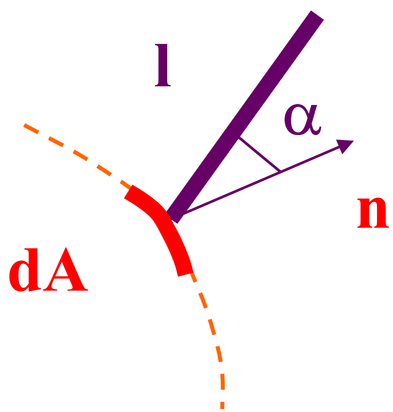
Time = t+dt
Length = l
Area = dA
Volume= l.dA
Fluid vel.: Vf
Boundary vel.:Vb

Generalizing…

- **For each area element there is a mass flow crossing it:**

$$d\dot{m} = \mathrm{Lim}\left(\frac{m^{t+\delta t} - m^{t}}{\delta t}\right) = \frac{(\rho l dA)^{t+\delta t} - (\rho l dA)^{t}}{\delta t}$$

- **l must be orthogonal to the crossing area:**



Mass Flux

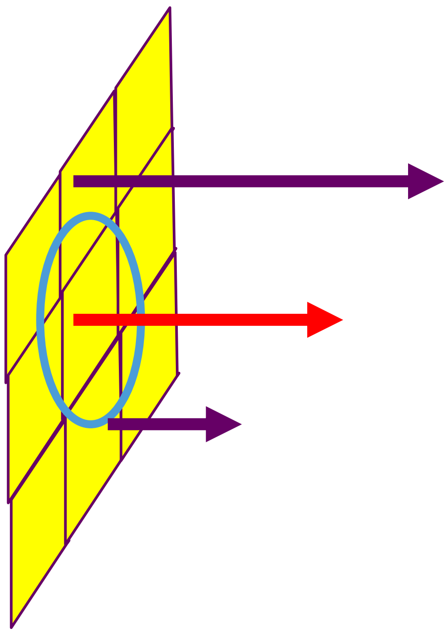
$$d\dot{m} = \frac{(\rho l \cos\alpha dA)^{t+\delta t} - (\rho l \cos\alpha dA)^{t}}{\delta t} \equiv \rho\overbrace{\left(\vec{n}\cdot\vec{V}_r\right)}dA$$

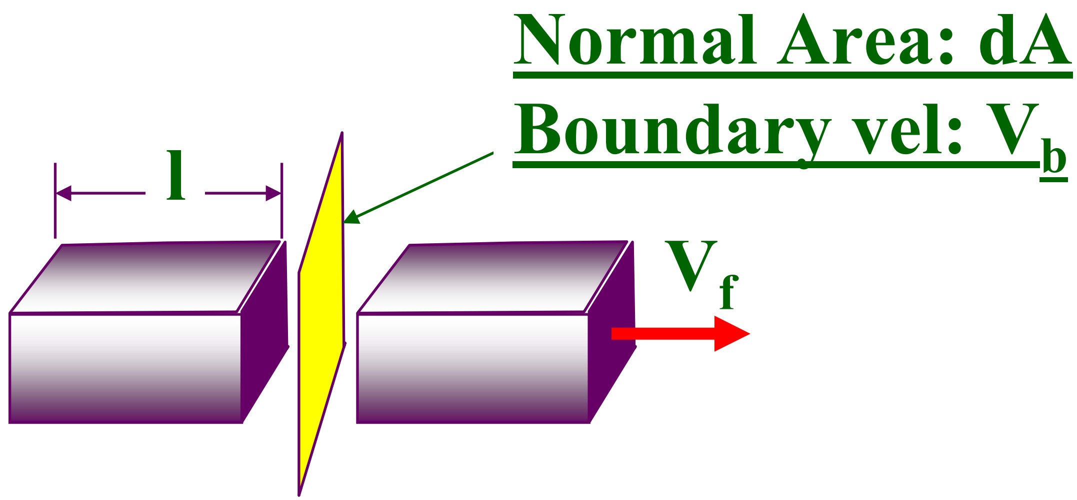
- **Vr is the relative velocity between the fluid and the boundary: $V_r = V_f - V_b$**

Mass Flow Rate: kg.sec^{-1}

- **Considering the area open to the flux the mass flow is then**

$$\dot{m} = \int d\dot{m} = \iint \rho\left(\vec{n}\cdot\vec{V}_r\right)dA$$





Flow rate of a generic variable β

$$\dot{\mathbf{B}} = \iint \beta\left(\vec{\mathbf{n}}\cdot\vec{\mathbf{V}}_{\mathbf{r}}\right)\mathbf{dA}$$

B flux: β.kg.sec^{-1}

$$\dot{\mathbf{M}} = \iint \rho\left(\vec{\mathbf{n}}\cdot\vec{\mathbf{V}}_{\mathbf{r}}\right)\mathbf{dA}$$

Mass flux: kg.sec^{-1}

$$\dot{\mathbf{U}} = \iint \mathbf{u}\rho\left(\vec{\mathbf{n}}\cdot\vec{\mathbf{V}}_{\mathbf{r}}\right)\mathbf{dA}$$

Internal Energy flux: J.sec^{-1}

$$\dot{\vec{\mathbf{X}}} = \iint \rho\left(\vec{\mathbf{n}}\cdot\vec{\mathbf{V}}_{\mathbf{r}}\right)\vec{\mathbf{V}}_{\mathbf{f}}\,\mathbf{dA}$$

Momentum flux: N

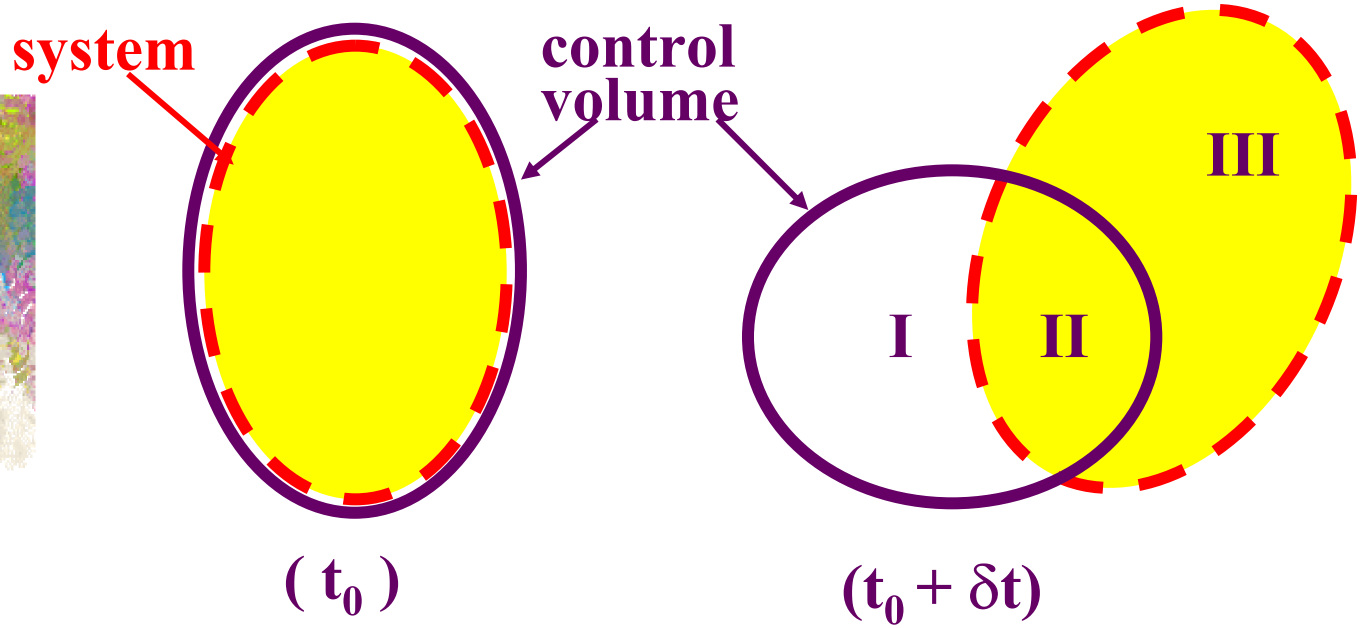
Reynolds Transport Theorem

- The control volume is a region of space bounded by the control surface which is deformable or not and where heat, work and mass can cross.

- The RTT translates the system time ratio in terms of the property ratio evaluated at a specific region on space – the control volume.

Reynolds Transport Theorem

- Let for an instant t_0 the control surface be coincident with the system boundary



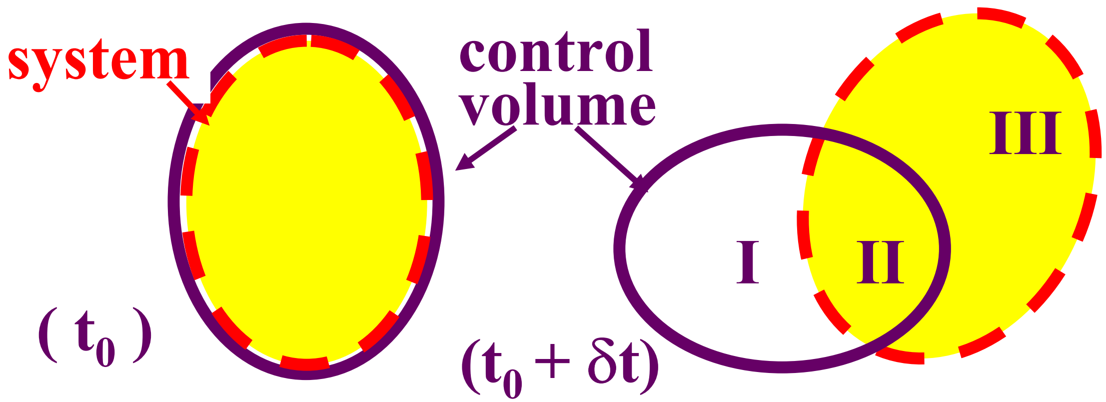
- At the instant $t_0+\delta t$ the system partially left the C.V. III is outside C.V.; II is still inside C.V. and I is filled by another system.

Reynolds Transport Theorem

The system time ratio written in terms of C.V. properties is:

$$\left.\frac{dB}{dt}\right|_{sys} = \lim_{\delta t \to 0}\left(\frac{B_{III}^{t+\delta t} + B_{II}^{t+\delta t} - B^{t}}{\delta t}\right)$$

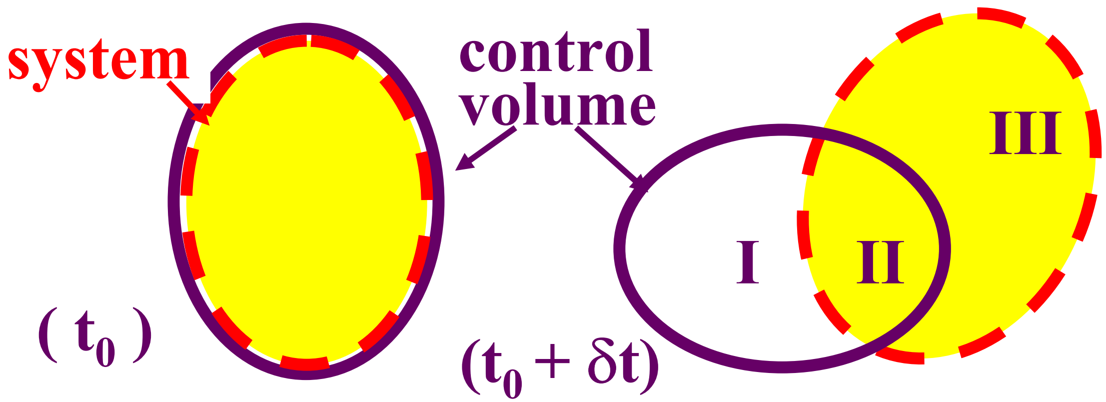
$$\equiv \lim_{\delta t \to 0}\left(\frac{B_{I}^{t+\delta t} + B_{II}^{t+\delta t} - B^{t}}{\delta t} + \frac{B_{III}^{t+\delta t}}{\delta t} - \frac{B_{I}^{t+\delta t}}{\delta t}\right)$$



Reynolds Transport Theorem

The first term is the time ratio of B within the C.V.:

$$\lim_{\delta t \to 0}\left(\frac{B_{I}^{t+\delta t} + B_{II}^{t+\delta t} - B^{t}}{\delta t}\right) \equiv \frac{d}{dt}\iiint_{vol} \beta \rho \, dV$$

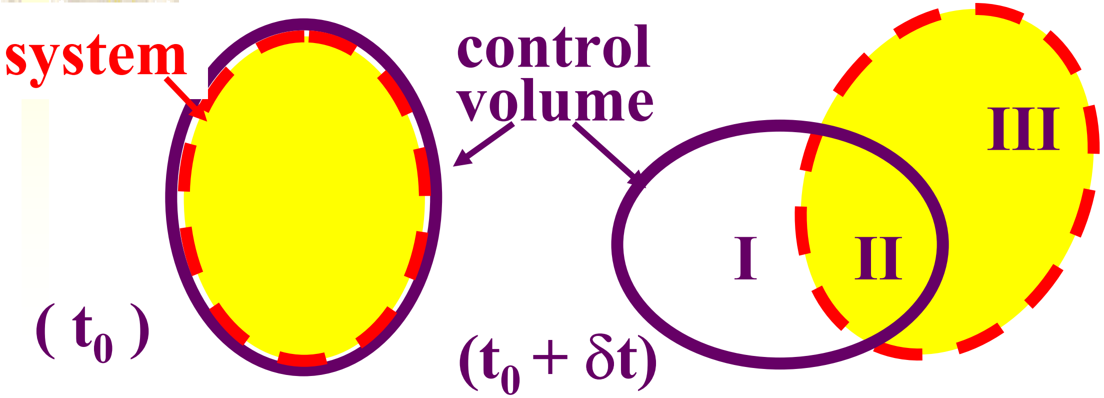


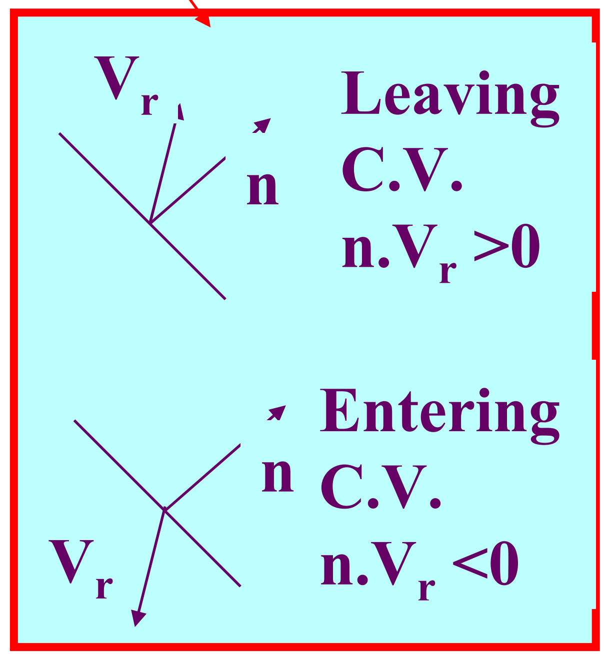
Reynolds Transport Theorem

The 2nd and 3rd terms represent the flux of B <u>out</u> and <u>in</u> of the C.V.:

$$\lim_{\delta t\to 0}\left(\frac{B_{III}^{t+\delta t}}{\delta t}-\frac{B_{I}^{t+\delta t}}{\delta t}\right)=\lim_{\delta t\to 0}\left(\frac{\delta t\cdot\iint_{III}\beta\rho\left(\vec{n}\cdot\vec{V}_r\right)dA}{\delta t}+\frac{\delta t\cdot\iint_{I}\beta\rho\left(\vec{n}\cdot\vec{V}_r\right)dA}{\delta t}\right)$$

$$=\oiint_{C.S.}\beta\rho\left(\vec{n}\cdot\vec{V}_r\right)dA$$





Reynolds Transport Theorem

- The system changes written in terms of a Control Volume,

$$\left.\frac{dB}{dt}\right|_{sys} = \frac{d}{dt}\iiint_{C.V.} \beta\rho dV + \oiint_{C.S.} \beta\rho\left(\vec{n}\cdot\vec{V}_r\right)dA$$

- The change of B in the system is equal to the change of B in the C.V. plus the net flux of B across the control surface.

- The lagrangian derivative of the system is evaluated for a region in space (fixed or not) by means of the RTT.

Transport Equations in Terms of Control Volume

- The Reynolds Transport Theorem is applied to the transport equations to express them by means of control volume properties

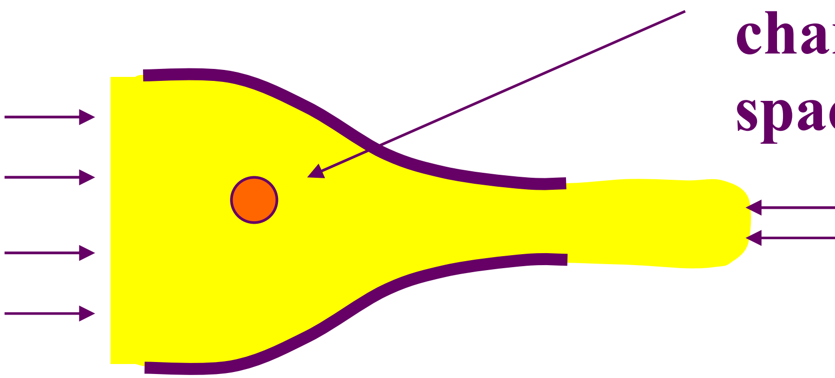
$$\left.\frac{dB}{dt}\right|_{sys} = \frac{d}{dt}\iiint_{C.V.} \beta\rho \, dV + \oiint_{C.S.} \beta\rho\left(\vec{n}\cdot\vec{V}_r\right)dA$$

Steady-flow assumption

Extensive and intensive properties within the control volume don't change with time, though they may vary with location.

Thus m_{CV}, E_{CV}, and V_{CV} are constant.

Pressure, temp, velocity do not change with time, but with space



Steady-flow assumption

- Observe that the time derivatives of the system and the C.V have different meanings:

$$\left.\frac{dB}{dt}\right|_{SYS} \neq \left.\frac{dB}{dt}\right|_{CV} \equiv \frac{d}{dt}\iiint_{vc} \rho\beta dV$$

- This allows the properties to vary from point-to-point but not with time, that is:

$$\left.\frac{d(M)}{dt}\right|_{CV} = \left.\frac{d(M\vec{V})}{dt}\right|_{CV} = \left.\frac{d(Me)}{dt}\right|_{CV} = 0$$

- However, material can still flow in and out of the control volume.
- The flow rate terms 'm' are not zero.

Mass Equation, β = 1 (scalar eq.)

- It express a mass balance for the C.V.
- The mass change within the C.V. is equal to the flux of mass crossing the C.S.

$$\left.\frac{dM}{dt}\right|_{sys} = \frac{d}{dt}\iiint_{C.V.} \rho\, dV + \oiint_{C.S.} \rho\left(\vec{n}\cdot\vec{V}_r\right) dA = 0$$

- The integral form is too complex to evaluate.
- Assume ***uniform properties***, i.e, density and velocities at the inlets and outlets

$$\left.\frac{dM}{dt}\right|_{sys} = \frac{d(\rho\forall)}{dt} + \underbrace{\sum(\rho VA)_{out}}_{\dot{m}_{out}} - \underbrace{\sum(\rho VA)_{in}}_{\dot{m}_{in}} = 0$$

The Conservation of Mass

$$\dot{M}_{IN} - \dot{M}_{OUT} = \left.\frac{dM}{dt}\right|_{CV}$$

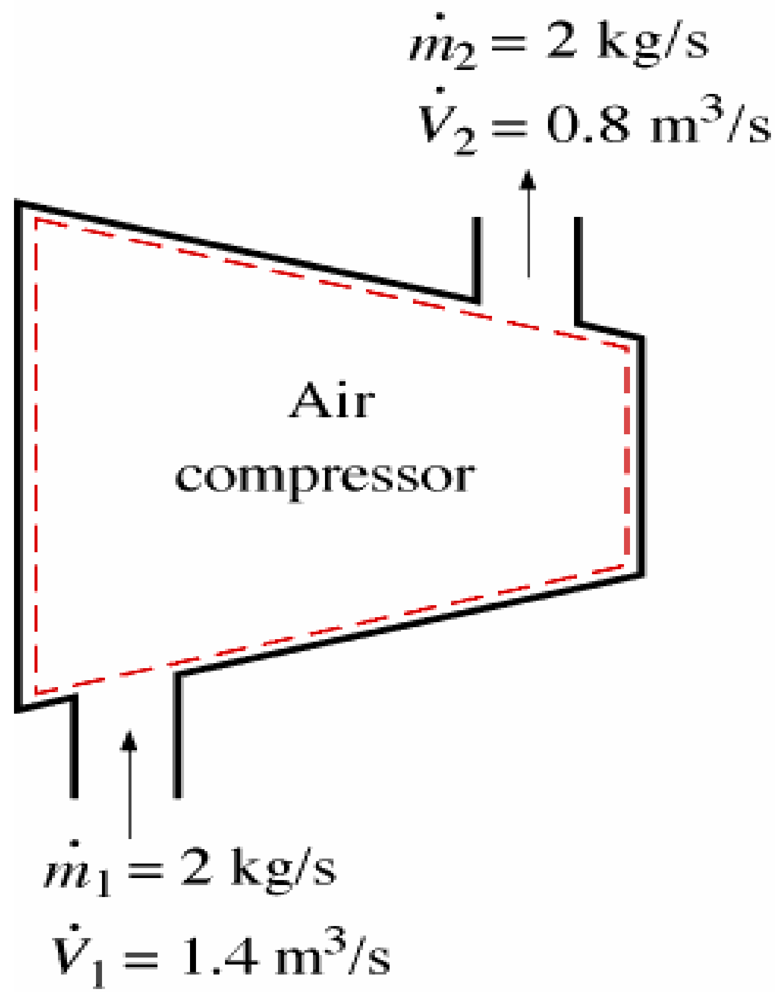
$$\begin{pmatrix} \textit{MASS FLOW} \\ \textit{RATE INTO} \\ \textit{C.V.} \end{pmatrix} - \begin{pmatrix} \textit{MASS FLOW} \\ \textit{RATE OUT OF} \\ \textit{C.V.} \end{pmatrix} = \begin{pmatrix} \textit{RATE OF CHANGE} \\ \textit{OF MASS IN THE} \\ \textit{C.V.} \end{pmatrix}$$

During Steady Flow Process, Volume Flow Rates are not Necessarily Conserved

- **Steady flow**
- **One inlet**
- **One outlet**

$$\dot{m}_1 = \dot{m}_2$$

$$\dot{V}_1 \neq \dot{V}_2$$



Momentum Equation, $\beta = V$, (vector eq., it has three components)

- **It express a force balance for the C.V. accordingly to <u>Newton's 2nd Law</u>.**

- **The momentum change within the C.V. is equal to the resultant force acting on the C.V.**

$$\left.\frac{d M\vec{V}}{dt}\right|_{sys} = \frac{d}{dt}\iiint_{C.V.} \rho \vec{V} dV + \oiint_{C.S.} \rho\left(\vec{n}\cdot\vec{V}_r\right)\vec{V}\, dA = \sum \vec{F}_{ext}\begin{bmatrix}\text{gravity}\\ \text{presure}\\ \text{shear stress}\end{bmatrix}$$

Momentum Equation, $\beta = V$, (vector eq., it has three components)

Constituting the external forces,

$$\frac{d}{dt}\iiint_{C.V.} \rho \vec{V} dV + \oint\!\!\oint_{C.S.} \rho\left(\vec{n}\cdot\vec{V}_r\right)\vec{V}\, dA = \iiint_{C.V.} \rho \vec{g} dV + \oint\!\!\oint_{C.S.} \left(-\vec{n}\cdot P\right) dA + \oint\!\!\oint_{C.S.} \left(\vec{n}\cdot\tau\right) dA$$

- The gravity force acts on the volume.
- The pressure force is a normal force acting inward at the C.S.
- The shear force acts tangentially at the C.S.

Momentum Equation, β = V, (vector eq., it has three components)

- **Assuming uniform properties: density and velocities (inlets/outlets)**
- **Neglecting the shear forces**

$$\frac{d\left(\rho\vec{V}\forall\right)}{dt}+\sum\left(\dot{m}\vec{V}_f\right)_{out}-\sum\left(\dot{m}\vec{V}_f\right)_{in}=\rho\vec{g}\Delta\forall+\oiint_{C.S.}\left(-\vec{n}\cdot P\right)dA$$

The Conservation of Momentum
- Newton 2nd Law -
Two Ports C.V. (one inlet/one outlet)

$$\left.\frac{d M\vec{V}}{dt}\right|_{CV} + \dot{M}\left(\vec{V}_{OUT} - \vec{V}_{OUT}\right) = \sum \vec{F}_{EXT}$$

$$\begin{pmatrix}\text{RATE OF CHANGE}\\ \text{OF MOMENTUM}\\ \text{IN THE C.V.}\end{pmatrix} - \begin{pmatrix}\text{MOMENTUM}\\ \text{FLUX IN}\\ \text{TOTHE C.V.}\end{pmatrix} + \begin{pmatrix}\text{MOMENTUM}\\ \text{FLUX OUT}\\ \text{TO THE C.V.}\end{pmatrix} = \begin{pmatrix}\text{NET FORCE}\\ \text{ACTING}\\ \text{ON THE C.V.}\end{pmatrix}$$

Energy Equation, β = e, (scalar eq.)

- It express the energy balance for the C.V.
- The momentum change within the C.V. is equal to the resultant force acting on the C.V.

$$\left.\frac{dMe}{dt}\right|_{sys} = \frac{d}{dt}\iiint_{C.V.} \rho e\, dV + \oiint_{C.S.} \rho\left(\vec{n}\cdot\vec{V}_r\right) e\, dA = \frac{dQ}{dt} - \frac{dW}{dt}$$

Energy Equation, β = e, (scalar eq.)

- The integral form is dropped. We will launch a lumped analysis with uniform properties.
- The energy equation becomes:

$$\frac{d(\rho e \forall)}{dt} + \sum(\dot{m}e)_{out} - \sum(\dot{m}e)_{in} = \frac{dQ}{dt} - \frac{dW}{dt}$$

- The heat and work convention signs for system holds for C.V.:

1. Heat IN and Work OUT to C.V. are (+)
2. Heat OUT and Work IN to C.V. are (-)

Let's look at the heat transfer terms first:

We want to combine them into a single term to give us the net heat transfer

$$\dot{Q}_{net} = \dot{Q}_{in} - \dot{Q}_{out}$$

For simplicity, we'll drop the "net" subscript

$$\dot{Q} = \dot{Q}_{net}$$

We'll do the same thing with work

Work involves boundary, shaft, electrical, and others

$$\dot{W} = -\dot{\mathbf{W}}_{\mathbf{in}} + \dot{\mathbf{W}}_{\mathbf{out}}$$

steady state regime and a two port (one inlet/one outlet) C.V. the energy equation reduces to:

$$\dot{m}\left(e_{out} - e_{in}\right) = \dot{Q} - \dot{W}$$

$$\begin{pmatrix} \text{FLUX} \\ \text{OF ENERGY} \\ \text{OUT TO THE C.S.} \end{pmatrix} - \begin{pmatrix} \text{FLUX} \\ \text{OF ENERGY} \\ \text{IN TO THE C.S.} \end{pmatrix} = \begin{pmatrix} \text{NET HEAT} \\ \text{AND WORK} \\ \text{ON THE C.S.} \end{pmatrix}$$

Energy Equation, β = e, (scalar eq.)

To constitute the energy equation is necessary now establish:

1- The specific energy terms, ‘e’

2 – Split the work terms in *pressure work or flow work* (PdV) plus other type of work modes

The Specific Energy 'e'

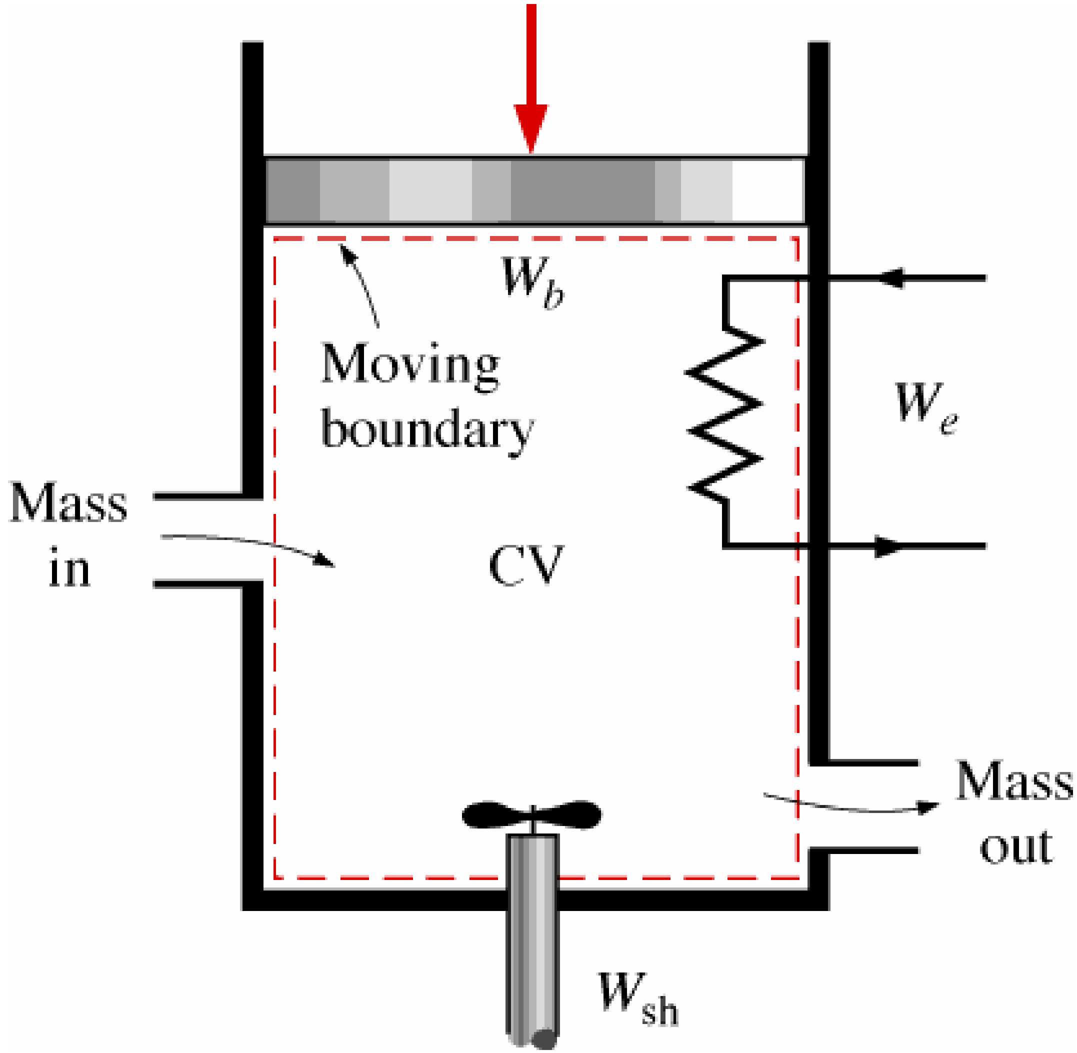
We will consider the specific energy the contribution of the:

1. fluid internal energy,
2. potential energy and
3. kinetic energy:

$$e = u + gz + \frac{V_I^2}{2}$$

Where V_I stands for the fluid velocity as seen from an inertial frame of reference.

Control Volume May Involve Boundary, Electrical, Shaft, and other Work



The breakup of the work term:

- **Work includes, in the general case, shaft work, such as that done by moving turbine blades or a pump impeller;**
- **the work due to movement of the CV surface or boundary work (usually the surface does not move and this is zero);**
- **the work due to magnetic fields, surface tension, etc., if we wished to include them (usually we do not); and**
- ***the work to move material in and out of the CV.***

Breakup of work, continued.

- **We are interested in breaking up work into two terms:**

1. **The work done on the CV by the increment m_i of mass as it enters and by the increment m_e of mass as it exits**

2. **All other works, which will usually just be shaft work, and which we will usually symbolize as W_{shaft} or just W.**

We normally split work into two terms:

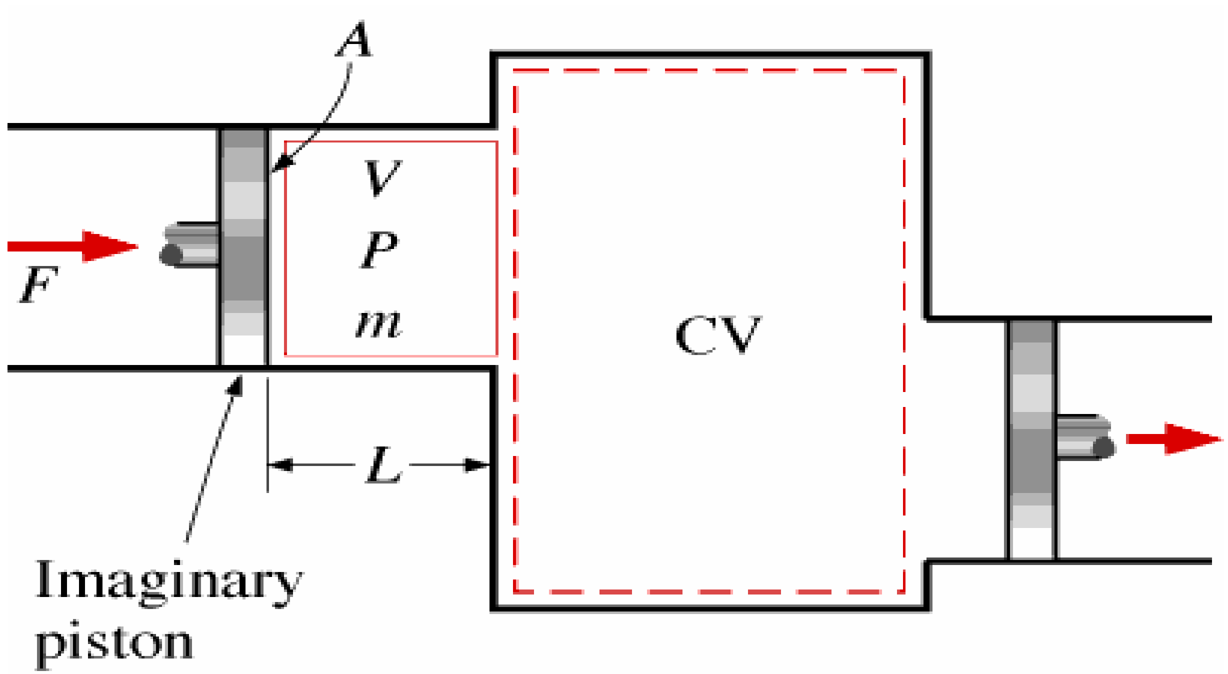
$$\dot{W} = \dot{W}_{FLOW} + \dot{W}_{SHAFT}$$

$$\dot{W}_{FLOW} = \textit{work done moving fluid in / out of c.v.}$$

$$\dot{\mathrm{W}}_{\mathrm{SHAFT}} = \text{net shaft work \& other types}$$

Schematic for Flow Work

Think of the slug of mass about to enter the CV as a piston about to compress the substance in the CV



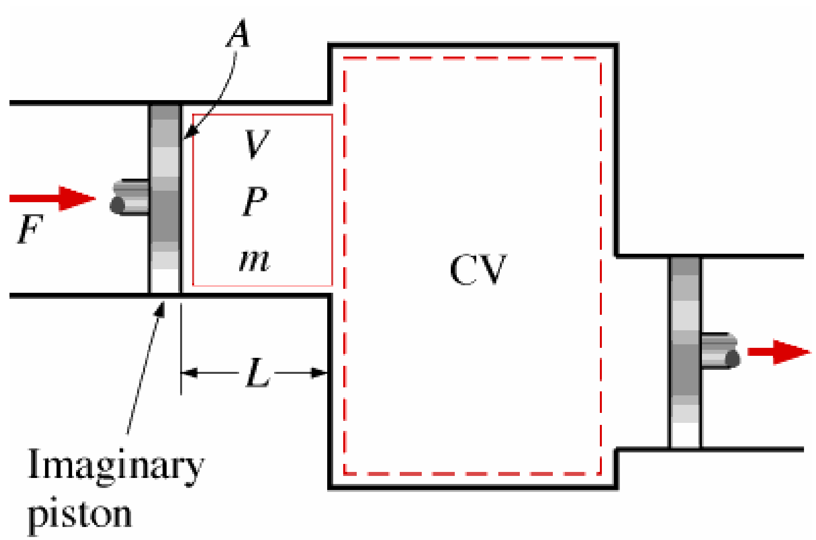
Schematic for Flow Work

The flow work is: $\Delta W_f = P\Delta V$

and the rate: $$\dot{W}_f = P\frac{d(\Delta V)}{dt} = P\left(\vec{n}\cdot\vec{V}_r\right)A = \frac{P}{\rho}\dot{M}$$

Which is the volumetric work to push or pull the slug of mass in to the C.V.

The scalar product gives the right sign if the C.V. is receiving or giving work



Energy Equation

Replacing the definitions of ‘e’ and W_f into the energy equation:

$$\frac{d}{dt}\left[\rho\left(u+\frac{V_I^2}{2}\right)\forall\right]+$$

$$\sum\left[\left(u+\frac{V_I^2}{2}+gz+\frac{P}{\rho}\right)\dot{m}\right]_{OUT}-\sum\left[\left(u+\frac{V_I^2}{2}+gz+\frac{P}{\rho}\right)\dot{m}\right]_{IN}=\dot{Q}-\dot{W}_{shaft}$$

What do the terms mean?

$$\left.\frac{dE}{dt}\right|_{cv} - \dot{m}\left(u + \frac{P}{\rho} + \frac{V_I^2}{2} + gz\right)_{in} + \dot{m}\left(u + \frac{P}{\rho} + \frac{V_I^2}{2} + gz\right)_{out} = \dot{Q} - \dot{W}_{shaf}$$

Rate of change of energy in CV.

Rate at which energy is convected into the CV.

Rate at which energy is convected out of the CV.

Rates of heat and work inter-actions

A Note About Heat

- Heat transfer should not be confused with the energy transported with mass into and out of a control volume

- Heat is the form of energy transfer as a result of temperature difference

Energy Equation

Remember the ENTALPY definition?

$$h = u + P/\rho$$

Lets use it in the Energy Equation!

$$\frac{d}{dt}\left[\rho\left(u+\frac{V_I^2}{2}\right)\forall\right]+$$

$$\sum\left[\left(h+\frac{V_I^2}{2}+gz\right)\dot{m}\right]_{OUT}-\sum\left[\left(h+\frac{V_I^2}{2}+gz\right)\dot{m}\right]_{IN}=\dot{Q}-\dot{W}_{shaft}$$

The energy equation can be simplified even more…..

Divide through by the mass flow:

$$q = \frac{\dot{Q}}{\dot{m}}$$ Heat transfer per unit mass

$$w_{shaft} = \frac{\dot{W}_{shaft}}{\dot{m}}$$ Shaft work per unit mass

We get the following for the <u>Steady State</u> Energy Equation in a Two Port C.V.

$$q - w_{shaft} = \left(h_{out} - h_{in}\right) + \left(\frac{V_{out}^2}{2} - \frac{V_{in}^2}{2}\right) + g(z_{out} - z_{in})$$

where z_{out} or z_{in} mean the cote at the *out* and *in* C.V. ports

Or in short-hand notation:

$$q - w_{shaft} = \Delta h + \Delta ke + \Delta pe$$

SUMMARY OF THE C.V. EQUATIONS

$$\frac{d(\rho\forall)}{dt} - \underbrace{\sum(\rho V A)_{in}}_{\dot{m}_{in}} + \underbrace{\sum(\rho V A)_{out}}_{\dot{m}_{out}} = 0$$

$$\frac{d(\rho\vec{V}\forall)}{dt} - \sum\left(\dot{m}\vec{V}_f\right)_{in} + \sum\left(\dot{m}\vec{V}_f\right)_{out} = \rho\vec{g}\Delta\forall + \iint_{C.S.}(-\vec{n}\cdot P)dA$$

$$\frac{d}{dt}\left[\rho\left(u+\frac{V_I^2}{2}\right)\forall\right] - \sum\left[\left(u+\frac{V_I^2}{2}+gz+\frac{P}{\rho}\right)\dot{m}\right]_{IN} + \sum\left[\left(u+\frac{V_I^2}{2}+gz+\frac{P}{\rho}\right)\dot{m}\right]_{OUT} = \dot{Q}-\dot{W}_{shaft}$$

$$\frac{d(\rho s\forall)}{dt} - \sum(\dot{m}s)_{in} + \sum(\dot{m}s)_{out} = \sum_{C.S.}\frac{\dot{Q}}{T} + \dot{S}_{gen}$$

RATE OF CHANGE INSIDE C.V.

FLUX IN THRU THE C.S.

FLUX OUT THRU THE C.S.

SOURCE TERMS

- **Problem 5.9** The water tank is filled through valve 1 with V1 = 10ft/s and through valve 3 with Q = 0.35 ft3/s. Determine the velocity through valve 2 to keep a constant water level.

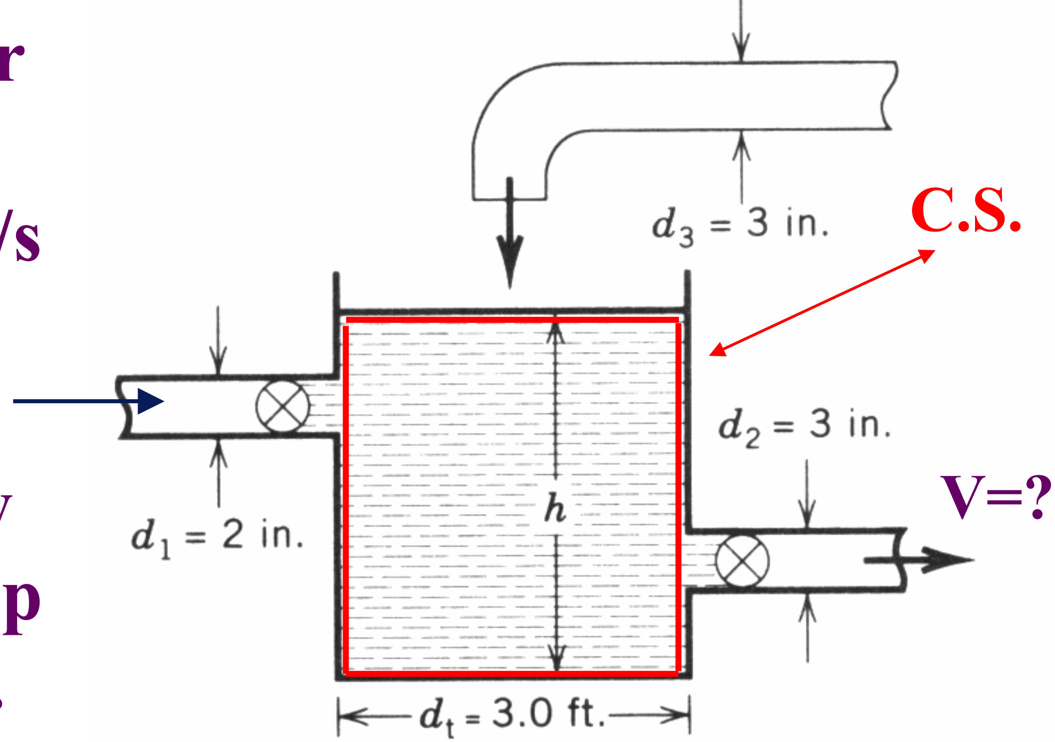


Figure P5-9 Water distribution tank.

$$(\rho VA)_2 - (\rho VA)_1 - (\rho VA)_3 = 0$$

$$\therefore \quad V_2 = \frac{V_1 d_1^2 + V_3 d_3^2}{d_2^2}$$

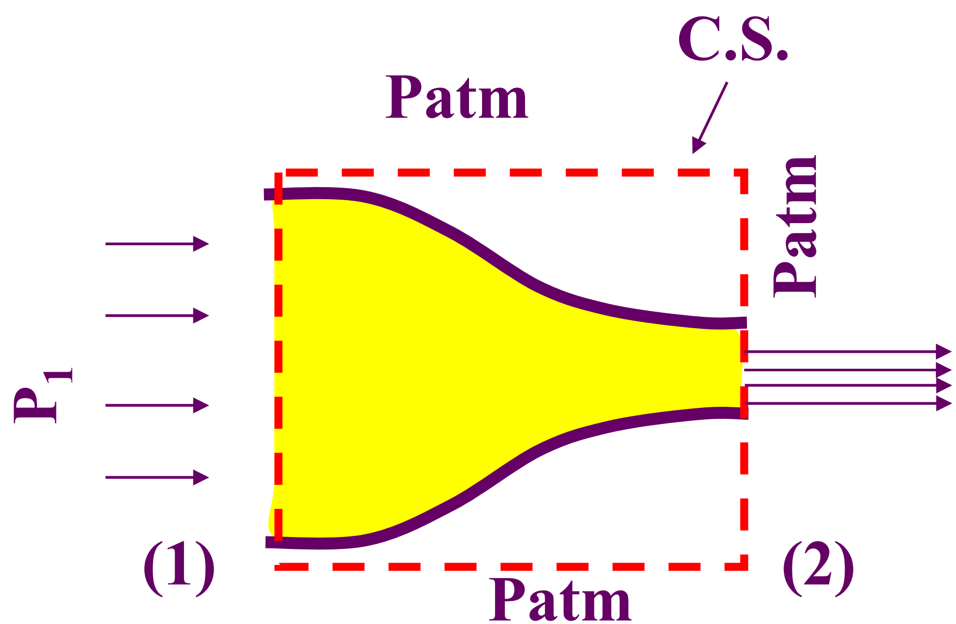
Steady and Unsteady Flow

- **Thermodynamic processes involving control volumes can be considered in two groups: *steady-flow processes* and *unsteady-flow processes*.**
- **During a *steady-flow process*, the fluid flows through the control volume steadily, experiencing no change with time at a fixed position. The mass and energy content of the control volume remain constant during a steady-flow process.**

Nozzle Reaction Force

The control surface bounds the nozzle (solid) plus the fluid. Every time the C.S. cross a solid there may be a mechanical force due to reaction.

Consider the inlet and outlet nozzle diameters as d_1 and d_2

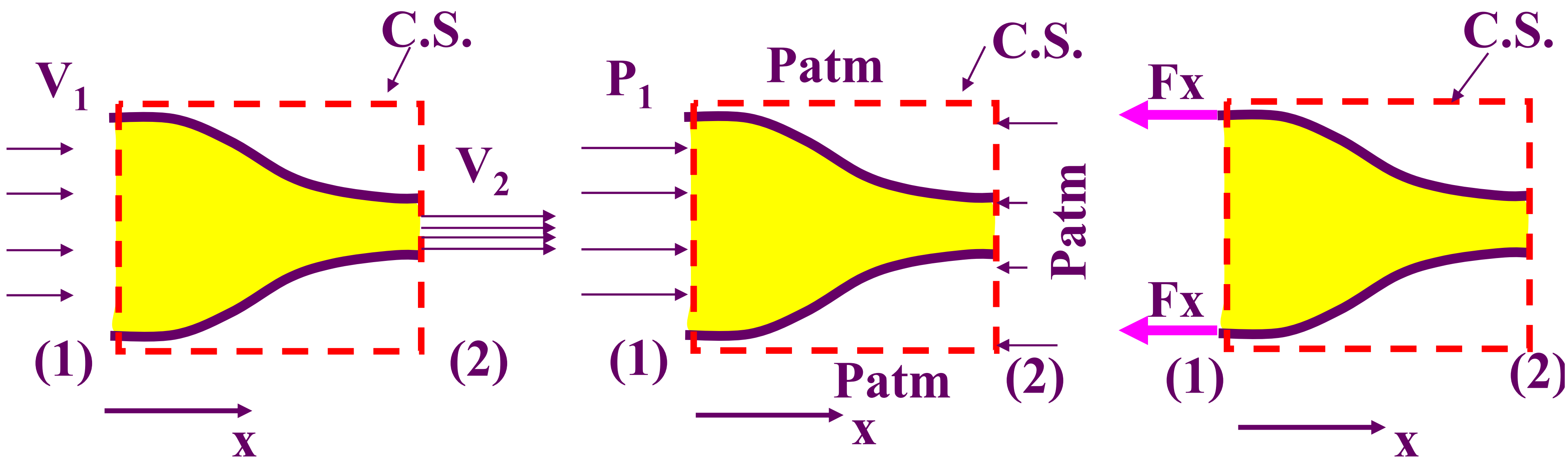


For steady state, d/dt = 0 and from mass conservation,

$\rho V_1 d_1^2 = \rho V_2 d_2^2$ ✸ $V_2 = V_1 (d_1/d_2)^2$ and $m = \rho V_1 \pi d_1^2/4$

Nozzle Reaction Force
(Vector equation → x component)

$$\left(\dot{m}\vec{V}_f\right)_{out} - \left(\dot{m}\vec{V}_f\right)_{in} = + \underset{C.S.}{\oiint}\left(-\vec{n}\cdot P\right)dA + \vec{F}_x$$



$$\dot{m}\left(V_2 - V_1\right) = \left(P_1 - P_{atm}\right)\cdot\frac{\pi d_1^2}{4} + F_x$$

2^{nd} Law Equation, $\beta = s$, (scalar eq.)

- It express the entropy transport by the mean flow field

$$\left.\frac{dMs}{dt}\right|_{sys} = \frac{d}{dt}\iiint_{C.V.} \rho s dV + \iint_{C.S.} \rho\left(\vec{n}\cdot\vec{V}_r\right)s\, dA = \iint_{C.S.} \frac{q}{T} dA + \frac{dS_{gen}}{dt}$$

Where

1. q is the local heat flux per unit area, that is in W/m^2, and
2. Sgen is the entropy generation term due to the Irreversibilities , Sgen ≥ 0

2^{nd} Law Equation, $\beta = s$, (scalar eq.)

- For uniform properties the integral forms can be dropped in favor of simple forms:

$$\frac{d\left(\rho s \forall\right)}{dt} - \sum\left(\dot{m}s\right)_{in} + \sum\left(\dot{m}s\right)_{out} = \sum_{C.S.} \frac{\dot{Q}}{T} + \dot{S}_{gen}$$

Where

1. q is the local heat flux per unit area, that is in W/m^2, and
2. Sgen is the entropy generation term due to the Irreversibilities , Sgen ≥ 0

Nozzle Reaction Force

Why is necessary two man to hold a fire hose?
Why to accelerate the water within the fire nozzle a reaction force appears?

Nozzle with adjustable throat diameter

100 Psi & 50 – 350 GPM