

## A Nonlinear Control Design for Tracked Robots with Longitudinal Slip

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**Abstract:** This paper presents an adaptive control strategy for a tracked mobile robot, in which the longitudinal slip of the left and right tracks are described by two unknown parameters. It is assumed that the kinematic model of the tracked robot is approximated by the one of a differential wheeled robot. An adaptive nonlinear feedback control law that compensates for the longitudinal slip is proposed to achieve a given trajectory tracking objective. Asymptotic stability of the close-loop system is ensured using an appropriate Lyapunov function. Numerical results show the benefits of the proposed approach.

Keywords: Mobile robots; Robot kinematics; Adaptive control; Robot control.

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### 1. INTRODUCTION

In the last years, the interest in tracked mobile robots has grown significantly because of the great variety of applications in unstructured environments as, for instance, forestry, mining, agriculture, military applications, search and rescue, hospital tasks and space exploration [Martínez et al., 2005]. However, all of these tasks require an efficient solution to the robot navigation problem.

The problem of tracking a reference trajectory is one of the most explored navigation problem in the robot literature [Fierro and Lewis, 1997]. The tracking problem consists in designing control inputs that stabilize the mobile robot about a trajectory generated by a reference model. Morin and Samson [2006] present an overview of recent tracking control methods for nonholonomic mobile robots. Tracked mobile robots are typical examples of non-holonomic systems. However, the motion control designs suitable for differential wheeled robots cannot be directly used for tracked robots [Martínez et al., 2005], unless the kinematic model of the tracked robot can be approximated by the one of a differential wheeled robot.

Locomotion based on tracks has a large ground contact patch that provides satisfactory stability and traction on various terrain conditions [Nourbakhsh and Siegwart, 2004]. Nevertheless, it is in general difficult to control tracked robots during applications in unstructured environment due to slip phenomena, which is an important factor that must be taken into account during the control design [Fan et al., 1995].

Many researches have investigated the slip phenomena in the navigation of mobile robots. Wang and Low [2008] give a general presentation on modeling of wheeled mobile robots in the presence of wheel skid and slip from the perspective of control design. Sidek and Sarkar [2008] provide a theoretical and systematic framework to include the slip into the overall system dynamics of wheeled mobile

robots. González et al. [2009a] present the synthesis of a control law for a wheeled mobile robot under slip condition using an LMI-based approach. Zhou et al. [2007] propose a nonlinear control law that uses an estimation of the slip obtained from the unscented Kalman filter (UKF). Other control designs that have also consider the slip can be found in Zhou and Han [2008] and González et al. [2009b].

In general, most of the proposed control techniques in the literature to deal with the slip assumes that the slip is available in real time. However, it is usually difficult to directly measure the slip and most techniques appeal to an estimator. Moreover, if the slip is not precisely estimated, for instance, due to sensor accuracy, the performance of the controllers can be seriously affected. Le et al. [1997] show that the slip of the tracks can be estimated from the robot pose using an extended Kalman filter (EKF). Song et al. [2008] present a nonlinear sliding mode observer for the estimation of tracked vehicle slip parameters based on the vehicle kinematic equations and sensor measurements. Ward and Iagnemma [2008] propose a model-based approach to estimating longitudinal wheel slip and detecting immobilized conditions of autonomous mobile robots operating on outdoor terrain. An experimental model used to describe the slip parameters of the kinematics model of a tracked mobile robot is presented in Moosavian and Kalantari [2008]. Others works that have also consider the slip estimation problem can be found in Angelova et al. [2006], Ojeda et al. [2006], Reina et al. [2008], Iagnemma and Ward [2009].

The main contribution of this paper is to design a controller that is able to compensate for the slip without estimating or measuring it. Inspired by Fukao et al. [2000], an adaptive controller which uses an update rule to compensate for the slip is proposed. Furthermore, the asymptotic stability of the closed-loop system is ensured using an appropriate Lyapunov function based on Kim and Oh [1998].

The paper is organized as follows. Section 2 presents the kinematic model of a tracked mobile robot under slip condition. Section 3 introduces the adaptive tracking control strategy that is able to compensate for the slip. Section 4 presents the numerical results using the proposed control technique. Finally, Section 5 presents the conclusions.

## 2. MODEL OF THE TRACKED MOBILE ROBOT

This section presents the kinematic model of a tracked mobile robot under slip condition. The longitudinal slip of the left and right tracks are described by two unknown parameters. It is assumed that the robot will operate at low speed, since the lateral slip is zero during straight line motion and it can be neglected when the robot turns on the spot [González et al., 2009a]. Moreover, the kinematic model of a tracked robot can only be approximated by the one of a differential drive wheeled robot if slow motions are assumed [Morales et al., 2009].

Fig. 1 shows the schematic model of a tracked mobile robot. It is assumed that the mobile robot can be represented by a rigid body with two independent tracks. The motion of the robot is described by its position  $(X, Y)$  and its orientation  $\psi$  in an inertial coordinate frame  $F_1(x_w, y_w)$ . The robot position is given by the coordinate of its geometric center  $C$ , which is also the origin of the local coordinate frame  $F_2(x_m, y_m)$ . The distance between the two tracks is  $b$ . Furthermore, the translation velocity is denoted by  $v$  and the rotational velocity by  $\omega = d\psi/dt$ .

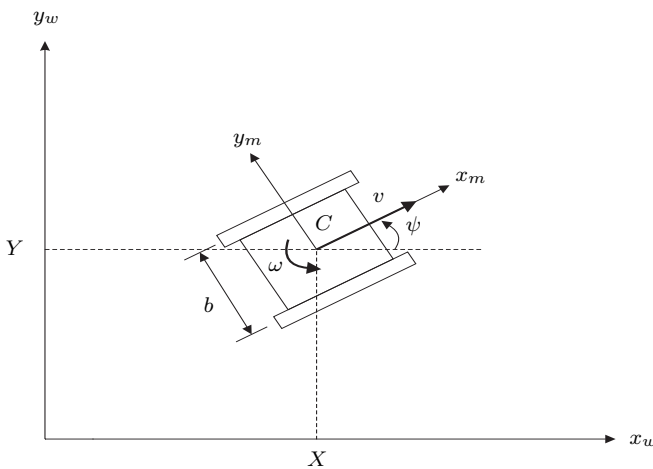


Fig. 1. Schematic model of the tracked mobile robot.

As shown in Zhou et al. [2007], the kinematic model of a tracked robot under slip condition is given by

$$\begin{pmatrix} \dot{X} \\ \dot{Y} \\ \dot{\psi} \end{pmatrix} = \begin{pmatrix} \frac{r(\omega_L p_L^{-1} + \omega_R p_R^{-1})}{2} \cos \psi \\ \frac{r(\omega_L p_L^{-1} + \omega_R p_R^{-1})}{2} \sin \psi \\ \frac{r(-\omega_L p_L^{-1} + \omega_R p_R^{-1})}{b} \end{pmatrix} \quad (1)$$

where  $q = (X, Y, \psi)^T$  denotes the pose (position and orientation) of the robot in the inertial frame  $F_1(x_w, y_w)$ ,  $r$  is the radius of the wheels, and  $\omega_L$  and  $\omega_R$  are respectively the left and right angular velocities of the wheels. The two unknown parameters  $p_L$  and  $p_R$  are defined as

$$p_L = \frac{1}{1 - i_L} \quad \text{and} \quad p_R = \frac{1}{1 - i_R}$$

with  $i_L$  and  $i_R$  denoting the longitudinal slip ratio of the left and right wheels, respectively, given by

$$i_L = \frac{(r\omega_L - v_L)}{r\omega_L}, \quad 0 \leq i_L < 1$$

$$i_R = \frac{(r\omega_R - v_R)}{r\omega_R}, \quad 0 \leq i_R < 1$$

where  $v_L$  and  $v_R$  are respectively the linear velocities of the left and right wheels with relation to the terrain.

## 3. ADAPTIVE TRACKING CONTROL

This section proposes an adaptive tracking control strategy for the kinematic model (1). The design is divided in three steps: first, a tracking control law is found by neglecting the slip; next, an update rule is designed to compensate for the slip; and finally, closed-loop stability is shown using an appropriate Lyapunov function.

To derive the control strategy, consider the auxiliary velocity control input  $\eta = (v, \omega)^T$  of the kinematic model (1) that has as effective velocity control input  $\xi = (\omega_L, \omega_R)^T$ . The effective input  $\xi$  is related to the auxiliary input  $\eta$  according to the equation  $\eta = T\xi$  given by

$$\begin{pmatrix} v \\ \omega \end{pmatrix} = \begin{pmatrix} \frac{r(\omega_L p_L^{-1} + \omega_R p_R^{-1})}{2} \\ \frac{r(-\omega_L p_L^{-1} + \omega_R p_R^{-1})}{b} \end{pmatrix} = T \begin{pmatrix} \omega_L \\ \omega_R \end{pmatrix} \quad (2)$$

with

$$T = r \begin{pmatrix} \frac{1}{2p_L} & \frac{1}{2p_R} \\ -\frac{1}{bp_L} & \frac{1}{bp_R} \end{pmatrix}$$

and the inverse relation  $\xi = T^{-1}\eta$  is given by

$$\begin{pmatrix} \omega_L \\ \omega_R \end{pmatrix} = \frac{1}{2r} \begin{pmatrix} 2p_L & -bp_L \\ 2p_R & bp_R \end{pmatrix} \begin{pmatrix} v \\ \omega \end{pmatrix} \quad (3)$$

Thus, the effective input  $\xi$  can always be obtained if an auxiliary input  $\eta$  exists and solves the tracking problem for the following kinematic model:

$$\begin{pmatrix} \dot{X} \\ \dot{Y} \\ \dot{\psi} \end{pmatrix} = \begin{pmatrix} \cos \psi & 0 \\ \sin \psi & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v \\ \omega \end{pmatrix} \iff \dot{q} = S(q)\eta \quad (4)$$

where  $q = (X, Y, \psi)^T$  is the robot configuration.

The control objective is to find an auxiliary input  $\eta$  for the tracked mobile robot such that

$$\lim_{t \rightarrow \infty} (q_r - q) = 0$$

where the robot configuration  $q = (X, Y, \psi)^T$  is given by (4) and the reference trajectory  $q_r = (X_r, Y_r, \psi_r)^T$  is generated using the kinematic model

$$\dot{q}_r = S(q_r)\eta_r$$

that is

$$\begin{pmatrix} \dot{X}_r \\ \dot{Y}_r \\ \dot{\psi}_r \end{pmatrix} = \begin{pmatrix} \cos \psi_r & 0 \\ \sin \psi_r & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v_r \\ \omega_r \end{pmatrix} \quad (5)$$

where  $\eta_r = (v_r, \omega_r)^T$  is a given constant reference input described by a linear velocity  $v_r > 0$  and an angular

velocity  $\omega_r$ . Once,  $\eta$  is designed, the control input  $\xi$  is given by (3).

To achieve the control objective, we define the tracking error  $e = (e_1, e_2, e_3)^T$  in the frame  $F_1(x_w, y_w)$  as

$$\begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix} = \begin{pmatrix} \cos \psi & \sin \psi & 0 \\ -\sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X_r - X \\ Y_r - Y \\ \psi_r - \psi \end{pmatrix} \quad (6)$$

The dynamics of the error  $e$ , obtained using (4), (5) and (6), is given by

$$\begin{pmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \end{pmatrix} = \begin{pmatrix} \omega e_2 + v_r \cos e_3 - v \\ -\omega e_1 + v_r \sin e_3 \\ \omega_r - \omega \end{pmatrix} \quad (7)$$

Neglecting the slip, the following control input

$$\begin{aligned} \omega &= \omega_r + \frac{v_r}{2} \left[ k_2 (e_2 + k_3 e_3) + \frac{1}{k_3} \sin e_3 \right] \\ v &= v_r \cos e_3 - k_3 e_3 \omega + k_1 e_1 \end{aligned} \quad (8)$$

with  $k_i > 0$  and  $v_r > 0$  drives the error signals  $e$  to zero. This can be shown using the following Lyapunov function based on Kim and Oh [1998]:

$$V_0(e) = \frac{1}{2} e_1^2 + \frac{1}{2} (e_2 + k_3 e_3)^2 + \frac{(1 - \cos e_3)}{k_2}$$

whose derivative  $\dot{V}_0(e)$  is

$$\dot{V}_0(e) = -k_1 e_1^2 - \frac{v_r}{2} k_2 k_3 (e_2 + k_3 e_3)^2 - \frac{v_r}{2k_2 k_3} \sin^2 e_3 \quad (9)$$

Note that, in the domain  $D = \{e \in \mathbb{R}^3 \mid -\pi < e_3 < \pi\}$ ,  $V_0(e)$  is continuously differentiable,  $V_0(0) = 0$ ,  $V_0(e)$  is positive definite and  $\dot{V}_0(e)$  is negative definite. Thus, the equilibrium  $e = 0$  is asymptotically stable and consequently  $q - q_r$  converge to zero.

To use equation (3), it is necessary the knowledge of the parameters  $p_L$  and  $p_R$ . In some cases, it can be assumed that the slip parameters can be measured (see Zhou et al. [2007]). However, it is in general difficult to precisely measure the slip. To overcome this difficulty, we propose an update rule to compensate for the slip without the need of actually measuring it. Equation (3), considering now the estimates  $\hat{p}_L = p_L + \tilde{p}_L$  and  $\hat{p}_R = p_R + \tilde{p}_R$ , is given by

$$\begin{pmatrix} \omega_L \\ \omega_R \end{pmatrix} = \frac{1}{2r} \begin{pmatrix} 2\hat{p}_L & -b\hat{p}_L \\ 2\hat{p}_R & b\hat{p}_R \end{pmatrix} \begin{pmatrix} v \\ \omega \end{pmatrix} \quad (10)$$

where  $\tilde{p}_L$  and  $\tilde{p}_R$  are respectively the estimation error of  $p_L$  and  $p_R$ .

To derive the update rule, it is necessary to rewrite (7) that depends on the auxiliary velocity (2) which in turns depends on the new control velocity (10). Thus, the derivative of the error  $\dot{e}$  is now given by

$$\begin{aligned} \dot{e}_1 &= \left(1 + \frac{\tilde{p}_L}{p_L}\right) \left(\frac{e_2}{b} + \frac{1}{2}\right) \left(v - \frac{b}{2}\omega\right) \\ &\quad + \left(1 + \frac{\tilde{p}_R}{p_R}\right) \left(\frac{e_2}{b} - \frac{1}{2}\right) \left(v + \frac{b}{2}\omega\right) + v_r \cos e_3 \\ \dot{e}_2 &= \left(1 + \frac{\tilde{p}_L}{p_L}\right) \left(v - \frac{b}{2}\omega\right) \frac{e_1}{b} \\ &\quad - \left(1 + \frac{\tilde{p}_R}{p_R}\right) \left(v + \frac{b}{2}\omega\right) \frac{e_1}{b} + v_r \sin e_3 \\ \dot{e}_3 &= \omega_r + \frac{1}{b} \left(1 + \frac{\tilde{p}_L}{p_L}\right) \left(v - \frac{b}{2}\omega\right) \\ &\quad - \frac{1}{b} \left(1 + \frac{\tilde{p}_R}{p_R}\right) \left(v + \frac{b}{2}\omega\right) \end{aligned} \quad (11)$$

To obtain the update rule, we consider the following Lyapunov function candidate

$$V(e) = V_0(e) + \frac{\tilde{p}_L^2}{2\gamma_1 p_L} + \frac{\tilde{p}_R^2}{2\gamma_2 p_R}$$

with  $\gamma_i > 0$ . Assuming that the unknown parameters  $p_L$  and  $p_R$  are constant, and using (11), we obtain

$$\begin{aligned} \dot{V}(e) &= -k_1 e_1^2 - \frac{v_r}{2} k_2 k_3 (e_2 + k_3 e_3)^2 - \frac{v_r}{2k_2 k_3} \sin^2 e_3 \\ &\quad + \frac{\tilde{p}_L}{p_L} \left\{ \frac{\dot{\tilde{p}}_L}{\gamma_1} - \left(v - \frac{b}{2}\omega\right) \left[ \left(\frac{e_2}{b} + \frac{1}{2}\right) e_1 \right. \right. \\ &\quad \left. \left. - (e_2 + k_3 e_3) \frac{e_1}{b} - \frac{k_3 (e_2 + k_3 e_3)}{b} - \frac{\sin e_3}{bk_2} \right] \right\} \\ &\quad + \frac{\tilde{p}_R}{p_R} \left\{ \frac{\dot{\tilde{p}}_R}{\gamma_2} - \left(v + \frac{b}{2}\omega\right) \left[ -\left(\frac{e_2}{b} - \frac{1}{2}\right) e_1 \right. \right. \\ &\quad \left. \left. + (e_2 + k_3 e_3) \frac{e_1}{b} + \frac{k_3 (e_2 + k_3 e_3)}{b} + \frac{\sin e_3}{bk_2} \right] \right\} \end{aligned} \quad (12)$$

Now, choosing the update rule for  $\hat{p}_L$  and  $\hat{p}_R$  as

$$\begin{aligned} \dot{\hat{p}}_L &= \gamma_1 \left(v - \frac{b}{2}\omega\right) \left[ \left(\frac{e_2}{b} + \frac{1}{2}\right) e_1 \right. \\ &\quad \left. - (e_2 + k_3 e_3) \frac{e_1}{b} - \frac{k_3 (e_2 + k_3 e_3)}{b} - \frac{\sin e_3}{bk_2} \right] \\ \dot{\hat{p}}_R &= \gamma_2 \left(v + \frac{b}{2}\omega\right) \left[ -\left(\frac{e_2}{b} - \frac{1}{2}\right) e_1 \right. \\ &\quad \left. + (e_2 + k_3 e_3) \frac{e_1}{b} + \frac{k_3 (e_2 + k_3 e_3)}{b} + \frac{\sin e_3}{bk_2} \right] \end{aligned} \quad (13)$$

with  $v$  and  $\omega$  given by (8), equation (12) for  $\dot{V}(e)$  becomes

$$\begin{aligned} \dot{V}(e) &= -k_1 e_1^2 - \frac{v_r}{2} k_2 k_3 (e_2 + k_3 e_3)^2 - \frac{v_r}{2k_2 k_3} \sin^2 e_3 \\ &= \dot{V}_0(e) < 0 \end{aligned}$$

which is similar to (9) and consequently the equilibrium  $e = 0$  is asymptotically stable. However, the convergence of the estimated parameters to their true values are not guaranteed.

Thus, we have shown that if we choose the control input as (8) and (10) with the parameters update rules as (13) for the kinematic model (1) of the mobile robot

with the constant unknown parameters  $p_L$  and  $p_R$ , the equilibrium  $e = 0$  is asymptotically stable. Thus, the robot configuration  $q$  asymptotically converge to the reference configuration  $q_r$ .

Fig. 2 shows the schematic representation of the system composed of the reference trajectory, the adaptive kinematic controller and the robot. The numbering inside the blocks indicates the corresponding equation number.

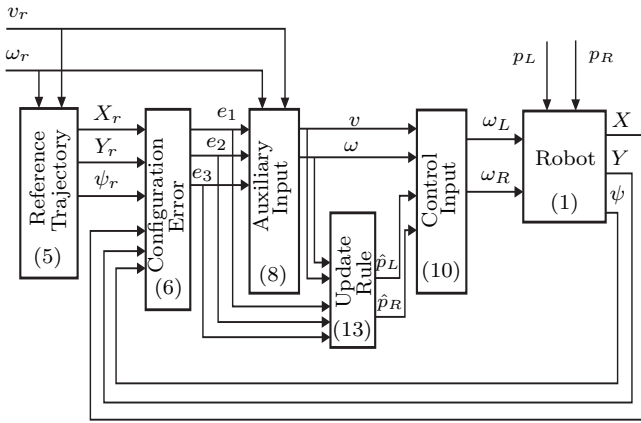


Fig. 2. Schematic representation of the close-loop system.

#### 4. NUMERICAL RESULTS

The physical parameters for the model of the robot, taken from Zhou et al. [2007], are given by  $b = 0.65$  m and  $r = 0.35$  m. The parameters of the adaptive controller are heuristically chosen as  $k_1 = 1$ ,  $k_2 = 20$  and  $k_3 = 1$ , and the parameters of the update rule are chosen as  $\gamma_1 = \gamma_2 = 20$ . The initial conditions of the update rule are taken as  $\hat{p}_L(0) = 1.0$  and  $\hat{p}_R(0) = 1.2$ . The total time for the computer simulation is  $t = 100$  s.

Although the unknown parameters are assumed constants during the control design, to demonstrate the performance of the close-loop system, the unknown parameters  $p_L$  and  $p_R$  are given by

$$\begin{aligned}
 0s \leq t < 20s : & \quad p_L = 2.00 \quad \text{and} \quad p_R = 1.00 \\
 20s \leq t < 30s : & \quad p_L = 1.00 \quad \text{and} \quad p_R = 1.00 \\
 30s \leq t < 40s : & \quad p_L = 1.00 \quad \text{and} \quad p_R = 2.50 \\
 40s \leq t < 50s : & \quad p_L = 1.50 \quad \text{and} \quad p_R = 2.50 \\
 50s \leq t < 60s : & \quad p_L = 1.50 \quad \text{and} \quad p_R = 1.00 \\
 60s \leq t < 75s : & \quad p_L = 1.00 \quad \text{and} \quad p_R = 1.00 \\
 75s \leq t < 85s : & \quad p_L = 1.00 \quad \text{and} \quad p_R = 1.50 \\
 85s \leq t : & \quad p_L = 1.00 \quad \text{and} \quad p_R = 1.00
 \end{aligned}$$

To generate the reference trajectory, model (5) is used with the initial condition  $q_r(0) = (0, 0, 0)^T$  and inputs  $v_r$  and  $w_r$  taken from Fukao et al. [2000]:

$$\begin{aligned}
 0s \leq t < 5s : & \quad v_r = 0.25 \left( 1 - \cos \frac{\pi t}{5} \right) \\
 & \quad w_r = 0 \\
 5s \leq t < 20s : & \quad v_r = 0.5 \\
 & \quad w_r = 0
 \end{aligned}$$

$$\begin{aligned}
 20s \leq t < 25s : & \quad v_r = 0.25 \left( 1 + \cos \frac{\pi t}{5} \right) \\
 & \quad w_r = 0 \\
 25s \leq t < 30s : & \quad v_r = 0.15\pi \left( 1 - \cos \frac{2\pi t}{5} \right) \\
 & \quad w_r = -v_r/1.5 \\
 30s \leq t < 35s : & \quad v_r = 0.15\pi \left( 1 - \cos \frac{2\pi t}{5} \right) \\
 & \quad w_r = v_r/1.5 \\
 35s \leq t < 40s : & \quad v_r = 0.25 \left( 1 + \cos \frac{\pi t}{5} \right) \\
 & \quad w_r = 0 \\
 40s \leq t : & \quad v_r = 0.5 \\
 & \quad w_r = 0
 \end{aligned}$$

Fig. 3 shows the robot trajectory in the fixed frame  $F_1(x_w, y_w)$  obtained using our proposed adaptive controller (ADP) and using the controller without slip compensation (WSC) by directly applying the control input (8) into kinematic model (4). The dashed line, the solid line and the dashdot line stand respectively for the reference trajectory (RT), the robot trajectory obtained using the adaptive controller (ADP) and the robot trajectory obtained using the controller without slip compensation (WSC). The initial conditions of the robot, depicted by a circle, is given by  $q(0) = (1/2, -1/2, -\pi/6)^T$ . This figure shows that the robot trajectory using the adaptive controller is able to follow the reference trajectory, even if the slip parameters change during the motion. On the other hand, the robot trajectory using the WSC controller is not able to follow the reference trajectory when the slip occurs.

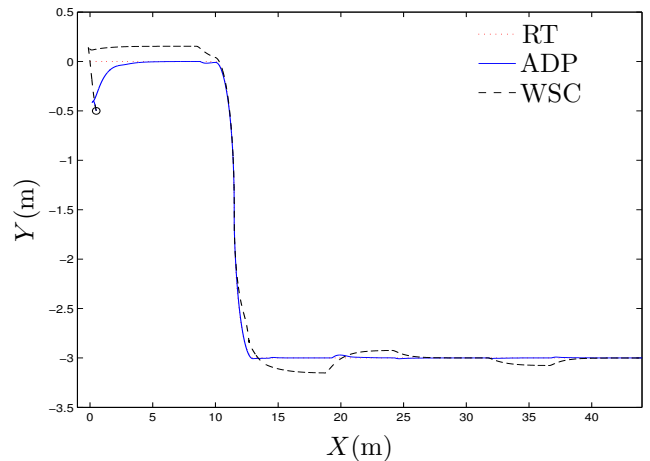


Fig. 3. Reference trajectory (RT), robot trajectory using the adaptive controller (ADP) and robot trajectory using the controller without slip compensation (WSC).

Fig. 4 shows the tracking errors  $e_1$ ,  $e_2$  and  $e_3$  in the fixed frame  $F_1(x_w, y_w)$ . The solid line stands for the error obtained using the adaptive controller (ADP), while the dashdot line stands for the error using the controller without slip compensation (WSC). As expected, the ADP controller achieves significantly better performance compared

to the WSC controller, whose graphs show an excessively large error whenever the slip occurs. On the other hand, the tracking error for the ADP controller is significantly smaller and only at the start it is large due to the robot initial condition.

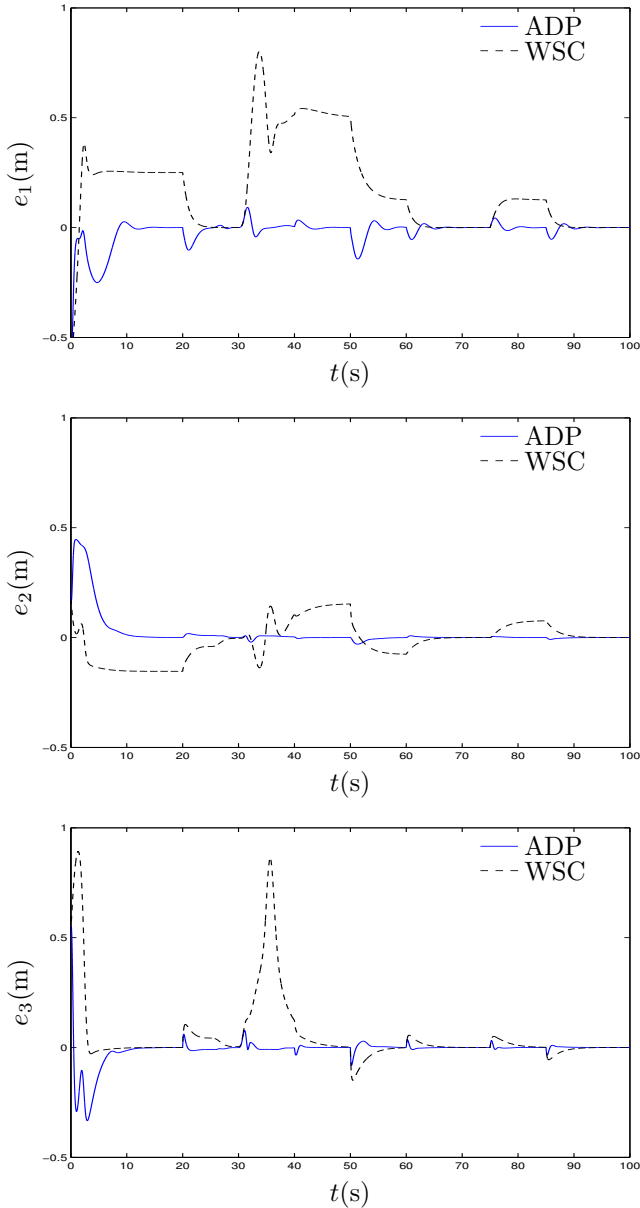


Fig. 4. The tracking error  $e = (e_1, e_2, e_3)^T$  obtained using the adaptive controller (ADP) and the controller without slip compensation (WSC).

Fig. 5 shows the parameter estimation using the ADP controller. The dashed line denotes the true values of the unknown parameters  $p_L$  and  $p_R$  and the solid line denotes the estimated values  $\hat{p}_L$  and  $\hat{p}_R$ . The high variation of the estimated parameters at the start is due to the large robot initial conditions in relation to the reference model. In this figure, the absence of slip on the left and right wheels is respectively represented as  $p_L = 1$  and  $p_R = 1$ .

Figs. 6 and 7 show for the ADP and WSC controllers, respectively, the effective velocities control input  $\omega_L$  (solid line) and  $\omega_R$  (dashed line). The two inputs  $\omega_L$  and  $\omega_R$  are independently applied to each wheel. It is worth to notice

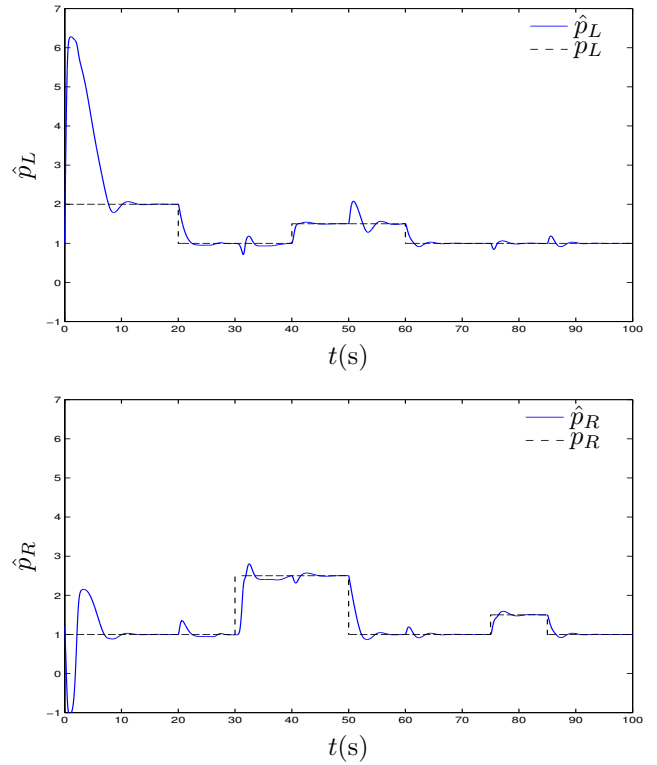


Fig. 5. True values of the unknown parameters  $p_L$  and  $p_R$  and the respective estimated values  $\hat{p}_L$  and  $\hat{p}_R$ .

that the the magnitude of the ADP control input in Fig. 6 is only slightly larger that the one obtained with the WSC controller in Fig. 7.

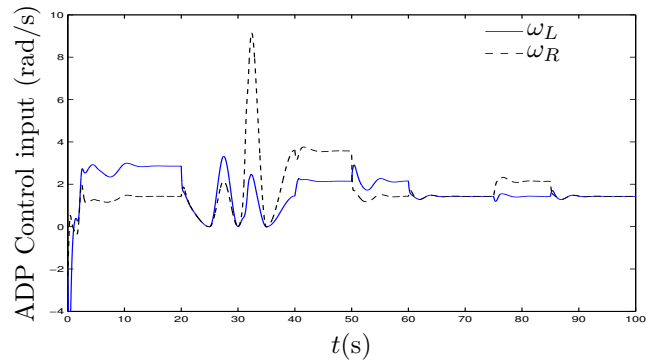


Fig. 6. Angular velocity of the right wheel  $\omega_R$  and and of the left wheel  $\omega_L$  using the ADP controller.

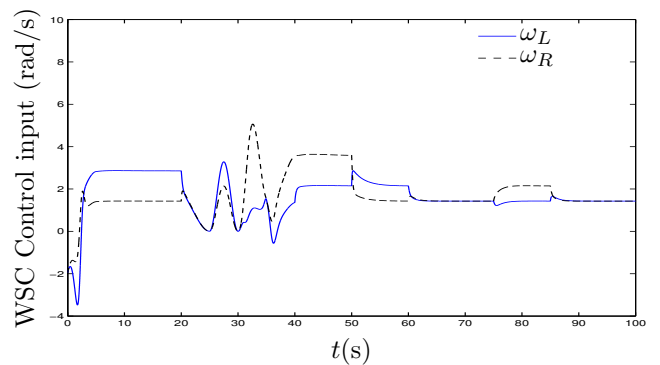


Fig. 7. Angular velocity of the right wheel  $\omega_R$  and of the left wheel  $\omega_L$  using the WSC controller.

## 5. CONCLUSIONS

This paper provides an adaptive control strategy for a tracked mobile robot with longitudinal slip. This control strategy is based on the kinematic model of the tracked robot, in which the longitudinal slip of the left and right tracks are described by two unknown parameters. A nonlinear feedback control law is proposed to achieve the trajectory tracking objective using an update rule that compensates for the slip. The stability of the closed-loop system is guaranteed using an appropriate Lyapunov function. Numerical results show that the proposed controller is able to ensure that the robot trajectory follows a given reference trajectory even when slip occurs, thus, outperforming techniques that can not consider the slip effect. The proposed control design procedure can be extended to include the dynamics of the tracked mobile robot.

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