ADAPTIVE TRACKING CONTROL OF TRACKED MOBILE ROBOTS WITH UNKNOWN SLIP PARAMETER

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Abstract— This paper presents the design of an adaptive control law that guarantees stability for a tracked mobile robot under unknown longitudinal slip condition. The kinematic model of the mobile robot is derived considering the slip as an unknown parameter. A control law that actuates on the angular velocities of the robot wheels is designed such that the robot follows a given reference trajectory. An update rule is used to estimate in real time the unknown slip parameter. The asymptotic stability of the global closed-loop system is ensured using an appropriate Lyapunov function. Numerical results show the usefulness of the proposed control strategy.

Keywords— Nonholonomic systems, mobile robot, kinematic model, adaptive control.

Resumo— Este trabalho apresenta o projeto de uma lei de controle adaptativo que garante estabilidade para um robô móvel acionado por rodas do tipo esteira sujeito a um deslizamento longitudinal desconhecido. O modelo cinemático do robô móvel é derivado considerando o deslizamento como um parâmetro incerto. Uma lei de controle que fornece as velocidades angulares das rodas do robô é projetada para que o robô siga uma dada trajetória de referência. Uma lei de adaptação é usada para estimar o parâmetro de deslizamento. A estabilidade assintótica do sistema em malha-fechada é assegurada usando-se uma função de Lyapunov apropriada. Resultados numéricos mostram a eficácia do método de controle proposto.

Palavras-chave— Sistemas não-holonômicos, robôs móveis, modelo cinemático, controle adaptativo.

1 Introduction

In recent years, the interest in mobile robots has grown significantly because of the great variety of applications in unstructured environments, where a high degree of autonomy is required. These applications usually require the robot to travel across off-road environments in tasks as forestry, mining, agriculture, army, etc (González et al., 2009). All of these tasks require an efficient solution to the robot navigation problem, which has received increasing attention due to its theoretical challenges.

One of the main navigation problem, the problem of tracking a reference trajectory, consists in designing control inputs that stabilize the mobile robot in a given reference trajectory. In general, this is a difficult problem, since mobile robots are typical examples of systems that has nonholonomic constraints (Kolmanovsky and McClamroch, 1995). According to Josephs and Huston (2002), if a system has constraint equations that involve velocities, accelerations, or derivatives of system coordinates, the constraint equations are said to be nonholonomic, or kinematic, and the mechanical system is said to be a nonholonomic system.

Many researchers investigate various tracking control designs (Dierks and Jagannathan, 2009; Lee et al., 2009; Michalek et al., 2009). Tracking control design for wheeled and for tracked mobile robots can be respectively found in Morin and Samson (2008) and in Fan et al. (1995). Although the kinematics model of the tracked robot are similar to the wheeled one, the former has a much larger ground contact patch, which is able to provide better stability and traction at various terrain conditions (Nourbakhsh and Siegwart, 2004).

Most control design techniques for mobile robots are based on the assumption that the wheels roll without slipping. However, the slip has a critical influence on the performance of mobile robots that cannot be neglected. Thus, to attain higher performance, it is necessary to incorporate the slip parameters into the model of the robot.

Many researches have addressed the slip phenomenon in the navigation of mobile robots (Matyukhin, 2007; Wang and Low, 2008; Sidek and Sarkar, 2008). However, in such works, the slip parameters are considered as disturbance or noise (Scaglia et al., 2009) or are estimated using some filtering technique (Zhou et al., 2007). Here, we propose an adaptive rule based in Fukao et al. (2000) to estimate the slip parameter.

In our paper, feedback velocity control inputs are designed, according to Gu and Hu (2006), for the kinematic steering system to enforce the position error converges to zero. Then, an update rule is designed such that the estimated slip parameter converge to the true slip parameter of the tracked robot. The update rule is derived using a Lyapunov function that guarantees the stability of the close-loop system.

The paper is organized as follows. In section 2, a kinematic model of a tracked mobile robot is derived, where the longitudinal slip is modeled by a unknown parameter. In section 3, an adaptive tracking controller is designed for the kinematic model and the stability of the proposed control system is analyzed using Lyapunov theory. The section 4 presents the results obtained by numerical simulations of the controlled system. Conclusions are presented in section 5.

2 Kinematic Model of a Tracked Mobile Robot with Slip

This section derives the kinematic model of a tracked robot with longitudinal slip. The slip is described by a unknown parameter, under the assumption that the robot will operate at low velocities. The lateral slip is zero for straight line motions, and it can be neglected when the vehicles turns "on the spot" or at low velocities. Figure 1 shows a model of the tracked mobile robot.

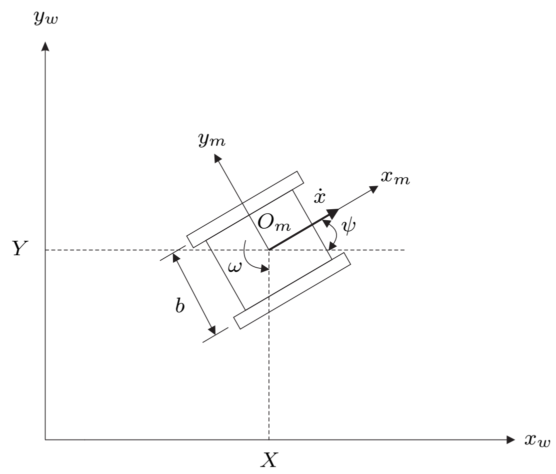

Figure 1: Model of the tracked mobile robot.

To describe the motion of the robot, its is necessary to define a fixed reference frame $F_1(x_w, y_w)$ and a moving frame $F_2(x_m, y_m)$ attached to the robot body with origin at its geometric center O_m. The motion of the robot is composed of the translation velocity $\dot{x}$ in the x_m-axis direction and the rotational velocity $d\psi/dt$ with ψ the yaw angle. Furthermore, the motion of the vehicle is constrained in the y_m-axis direction with $\dot{y} = 0$ (nonholonomic constraint).

The longitudinal slip ratio of the two wheels is defined as follows

$$i = \frac{(r\omega_L - v_L)}{r\omega_L} = \frac{(r\omega_R - v_R)}{r\omega_R}, \quad 0 \leq i < 1$$

where r is the radius of the wheels, ω_L and ω_R are respectively the angular velocities of the left and the right wheels. To simplify the notation, we redefine the slip parameter as

$$a = \frac{1}{(1-i)}$$

In the moving frame F_2, the model with longitudinal slip is given by

$$\begin{aligned} \dot{x} &= r(\omega_L + \omega_R)/2a \\ \dot{y} &= 0 \\ \dot{\psi} &= r(\omega_R - \omega_L)/ba \end{aligned}$$

where b is the distance between the two wheels.

Using some appropriate rotation matrix, from the reference frame F_2 to the reference frame F_1, the kinematic model can be written as

$$\begin{pmatrix} \dot{X} \\ \dot{Y} \\ \dot{\psi} \end{pmatrix} = \begin{pmatrix} r(\omega_L + \omega_R)\cos\psi/2a \\ r(\omega_L + \omega_R)\sin\psi/2a \\ r(\omega_R - \omega_L)/ba \end{pmatrix} \tag{1}$$

where $q = (X, Y, \psi)^T$ denotes the coordinates of the tracked vehicle in the inertial Cartesian frame F_1. The yaw angle ψ is assumed to be in $(-\pi, \pi]$.

The auxiliary control input η is defined as $\eta = (v, \omega)^T$ with $v = \dot{x}$ and $\omega = \dot{\psi}$. The effective control input u for the model (1) is defined as $u = (\omega_L, \omega_R)^T$. Note that η is related to u according to the following equation

$$\begin{pmatrix} v \\ \omega \end{pmatrix} = \begin{pmatrix} r(\omega_L + \omega_R)/2a \\ r(\omega_R - \omega_L)/ba \end{pmatrix} = T \begin{pmatrix} \omega_L \\ \omega_R \end{pmatrix} \tag{2}$$

with

$$T = \frac{r}{2a} \begin{pmatrix} 1 & 1 \\ -2/b & 2/b \end{pmatrix}$$

We have also that the effective control input $u = T^{-1}\eta$ is given by

$$\begin{pmatrix} \omega_L \\ \omega_R \end{pmatrix} = \frac{a}{2r} \begin{pmatrix} 2 & -b \\ 2 & b \end{pmatrix} \begin{pmatrix} v \\ \omega \end{pmatrix} \tag{3}$$

Substituting (3) in (1), we arrive to the following model

$$\dot{q} = S(q)\eta$$

that is

$$\begin{pmatrix} \dot{X} \\ \dot{Y} \\ \dot{\psi} \end{pmatrix} = \begin{pmatrix} \cos\psi & 0 \\ \sin\psi & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v \\ \omega \end{pmatrix} \tag{4}$$

Note that the nonholonomic constraint $\dot{y} = 0$ restrict the robot to move only in the direction normal to the axis of the driving tracks. This nonholonomic constraint can equivalently be written in the frame F_1 as

$$\begin{pmatrix} -\sin\psi & \cos\psi & 0 \end{pmatrix} \begin{pmatrix} \dot{X} \\ \dot{Y} \\ \dot{\psi} \end{pmatrix} = A(q)\dot{q} = 0$$

3 Adaptive Tracking Control

In this section, we consider the tracking control problem for the kinematic model (1) of tracked mobile robots with the slip given by the parameter i. The design is divided in three steps as follows: first, the tracking control law is found neglecting the slip; next, an update rule is designed to estimate the slip parameter; and finally, closed-loop stability is shown using an appropriate Lyapunov function.

In order to deal with the tracking control problem, we need to define the reference trajectory. The trajectory reference $q_r = (X_r, Y_r, \psi_r)^T$, in the fixed frame F_1, is generated using the kinematic model

$$\dot{q}_r = S(q_r)\eta_r$$

that is

$$\begin{pmatrix} \dot{X}_r \\ \dot{Y}_r \\ \dot{\psi}_r \end{pmatrix} = \begin{pmatrix} \cos\psi_r & 0 \\ \sin\psi_r & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v_r \\ \omega_r \end{pmatrix} \tag{5}$$

with $\eta_r = [v_r \quad \omega_r]^T$ containing the desired linear v_r and angular ω_r constant reference velocities. Note that the signals v_r and ω_r can not be simultaneously zero, otherwise, the reference trajectory q_r does not exist. The signal η_r in (5) is constructed to produce the desired motion. It is assumed that the signals η_r and $\dot{\eta}_r$ are bounded. This is not a severe restriction, since most practical reference trajectories satisfy this assumption.

The goal of the proposed methodology is to design an adaptive tracking controller for the tracked mobile robot with slipping such that

$$\lim_{t\to\infty} (q - q_r) = 0$$

where q is the robot configuration given by (1) and q_r is the reference trajectory given by (5).

To investigate the closed-loop stability, we define the error $e = (e_1, e_2, e_3)^T$ in the frame F_1 as

$$\begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix} = \begin{pmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X_r - X \\ Y_r - Y \\ \psi_r - \psi \end{pmatrix} \tag{6}$$

Then, the dynamics of the error e, derived using (4), (5) and (6), is given by

$$\begin{pmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \end{pmatrix} = \begin{pmatrix} \omega e_2 + v_r \cos e_3 - v \\ -\omega e_1 + v_r \sin e_3 \\ \omega_r - \omega \end{pmatrix} \tag{7}$$

Neglecting the slip, Gu and Hu (2006) showed that the control input

$$\begin{pmatrix} v \\ \omega \end{pmatrix} = \begin{pmatrix} v_r \cos e_3 + k_1 e_1 \\ \omega_r + v_r k_2 e_2 + k_3 \sin e_3 \end{pmatrix} \tag{8}$$

with $k_i > 0$ drives the error signals e to zero using the following Lyapunov function

$$V_0 = \frac{1}{2}\left(e_1^2 + e_2^2\right) + \frac{(1 - \cos e_3)}{k_2}$$

whose derivative is

$$\begin{aligned} \dot{V}_0 &= e_1\dot{e}_1 + e_2\dot{e}_2 + \dot{e}_3\frac{\sin e_3}{k_2} \\ &= -k_1 e_1^2 - \frac{k_3}{k_2}\sin^2 e_3 \leq 0 \end{aligned}$$

Now, if the slip parameter a that appear in (3) is unknown, we cannot choose directly the auxiliary control input as given by (8). Hence, we design an update rule to attain the control objective using estimate for a. Since a is not known, we use formula (3) considering now the estimate $\hat{a} = a + \tilde{a}$ with $\tilde{a}$ is the estimation error:

$$\begin{pmatrix} \omega_L \\ \omega_R \end{pmatrix} = \frac{\hat{a}}{2r} \begin{pmatrix} 2 & -b \\ 2 & b \end{pmatrix} \begin{pmatrix} v \\ \omega \end{pmatrix} \tag{9}$$

To derive the update rule, it is necessary to calculate (7) that depends on the auxiliary control input (2) which in turns depends on the new effective control input (9). Thus, the derivative of the error $\dot{e}$ is given by

$$\begin{aligned} \dot{e}_1 &= \frac{(a + \tilde{a})}{a}(e_2\omega - v) + v_r \cos e_3 \\ \dot{e}_2 &= -\frac{(a + \tilde{a})}{a} e_1\omega + v_r \sin e_3 \\ \dot{e}_3 &= \omega_r - \frac{(a + \tilde{a})}{a}\omega \end{aligned} \tag{10}$$

To obtain the update rule, we consider the following Lyapunov function candidate

$$V = V_0 + \frac{\tilde{a}^2}{2\gamma a} \tag{11}$$

with $a \geq 1$ and $\gamma > 0$. The derivative of V is given by

$$\dot{V} = e_1\dot{e}_1 + e_2\dot{e}_2 + \dot{e}_3\frac{\sin e_3}{k_2} + \frac{\tilde{a}}{\gamma a}\dot{\hat{a}}$$

substituting (10) in $\dot{V}$, we obtain

$$\dot{V} = \dot{V}_0 + \frac{\tilde{a}}{a}\left[\frac{\dot{\hat{a}}}{\gamma} - \left(v e_1 + \frac{\omega \sin e_3}{k_2}\right)\right] \tag{12}$$

Now, choosing the update rule for $\hat{a}$ as

$$\dot{\hat{a}} = \gamma\left(v e_1 + \frac{\omega \sin e_3}{k_2}\right) \tag{13}$$

with v and ω given by (8), the equation (12) for $\dot{V}$ take the form

$$\dot{V} = \dot{V}_0 = -k_1 e_1^2 - \frac{k_3}{k_2}\sin^2 e_3 \leq 0 \tag{14}$$

It is now possible to guarantee closed-loop stability by showing that $e = 0$ is an asymptotically stable equilibrium. Let the domain D be given by $D = \{e \in \mathbb{R}^3 \mid -\pi < e_3 < \pi\}$, then the Lyapunov function given in (11) is positive definite in $D - \{0\}$ with derivative $\dot{V} \leq 0$ in D. This implies that e_1, e_2 and the estimate parameter $\hat{a}$ are bounded. Since the reference velocity $\eta_r = (v_r, \omega_r)^T$ is bounded, we known from (8) that the auxiliary control input η is also bounded. Thus, $\dot{e}$ is bounded by (7). After all, $\ddot{V}(e, \dot{e})$ given by

$$\ddot{V} = -2k_1 e_1\dot{e}_1 - \frac{2k_3}{k_2}\sin e_3 \cos e_3 \dot{e}_3$$

is also bounded.

Since V is a nonincreasing function that converges to some constant value. Barbalat's Lemma (Li and Slotine, 1991; Khalil, 2001) shows that $\dot{V} \to 0$ as $t \to \infty$. Thus, from (14), we know that e_1 and e_3 tend to zero as $t \to \infty$.

It now remains to show that e_2 also converges to zero. Since we have already shown that all variables are bounded, closed-loop stability can be asserted by linearizing around the origin the augmented system $\dot{p} = [\dot{e} \quad \dot{\tilde{a}}]^T$ which contains the error equation (10) and the update law (13). Note that $\dot{\tilde{a}} = \dot{\hat{a}}$ since a is constant. Thus, the linearized model is given by

$$\dot{p} = A_p p$$

with

$$A_p = \begin{pmatrix} -k_1 & \omega_r & 0 & -v_r/a \\ -\omega_r & 0 & v_r & 0 \\ 0 & -k_2 v_r & -k_3 & -\omega_r/a \\ \gamma v_r & 0 & \gamma\omega_r/k_2 & 0 \end{pmatrix}$$

The characteristic equation for this linear system is readily obtained as

$$\alpha_4 s^4 + \alpha_3 s^3 + \alpha_2 s^2 + \alpha_1 s^1 + \alpha_0 = 0$$

with

$$\begin{aligned} \alpha_4 &= 1 \\ \alpha_3 &= k_1 + k_3 \\ \alpha_2 &= k_1 k_3 + \frac{\gamma}{a} v_r^2 + k_2 v_r^2 + \omega_r^2 + \frac{\gamma}{a k_2}\omega_r^2 \\ \alpha_1 &= k_1 k_2 v_r^2 + \frac{\gamma k3}{a} v_r^2 + \frac{\gamma k1}{a k_2}\omega_r^2 + k_3 \omega_r^2 \\ \alpha_0 &= \frac{\gamma k_2}{a} v_r^4 + \frac{2\gamma}{a} v_r^2 \omega_r^2 + \frac{\gamma}{a k_2}\omega_r^4 \end{aligned}$$

Since $\alpha_i > 0$, $\alpha_3\alpha_2 - \alpha_1\alpha_4 > 0$, $\alpha_1\alpha_2\alpha_3 - \alpha_0\alpha_3^2 - \alpha_1^2\alpha_4 > 0$ and $\alpha_0(\alpha_1\alpha_2\alpha_3 - \alpha_0\alpha_3^2 - \alpha_1^2\alpha_4) > 0$, the Routh-Hurwitz stability criterion (Gradshteyn and Ryzhik, 2000) ensures that all eigenvalues of A_p have negative real parts. Thus, the equilibrium $p = 0$ of the augmented system is asymptotically stable and consequently the system error e and the estimation error $\tilde{a}$ converge to zero as $t \to \infty$.

The next theorem summarizes our main results.

Theorem 1 *Consider the kinematic model* (1) *of the mobile robot with an unknown slip parameter given by a. If we choose the control input as* (8)-(9) *and the parameter update rule as* (13)*, then the equilibrium $e = 0$ is asymptotically stable. Consequently, the robot configuration q asymptotically follows the reference configuration q_r.*

4 Numerical Results

This section presents the numerical results using the proposed adaptive tracking control methodology.

The data for the mobile robot used in this section, taken from Zhou et al. (2007), are $b = 0.65$ m and $r = 0.35$ m. The control parameters for the controller are chosen as $k_1 = 6$, $k_2 = 8$ and $k_3 = 6$. The parameter for the adaptive rule is chosen as $\gamma = 10$. The initial conditions are taken as $q_r(0) = (0, 0, 0)^T$ and $\hat{a}(0) = 1$. Two reference trajectories are used. First, a linear trajectory generated with $v_r = 0.5$ m/s and $\omega_r = 0$ rad/s. Second, a circular trajectory generated with $v_r = 0.5$ m/s and $\omega_r = 0.25$ rad/s. The robot initial conditions for the linear and circular trajectory are respectively given by $q(0) = (0, -1, \pi/6)^T$ and $q(0) = (0, 1, \pi/4)^T$. To demonstrate the tracking performance, the unknown slip parameter changes from $i = 0$ to $i = 0.25$ at $t = 30$ s and from $i = 0.25$ to $i = 0$ at $t = 60$ s.

Figures 2 and 3 show the tracking error e in the fixed frame F_1 for the linear and circular references trajectories, respectively.

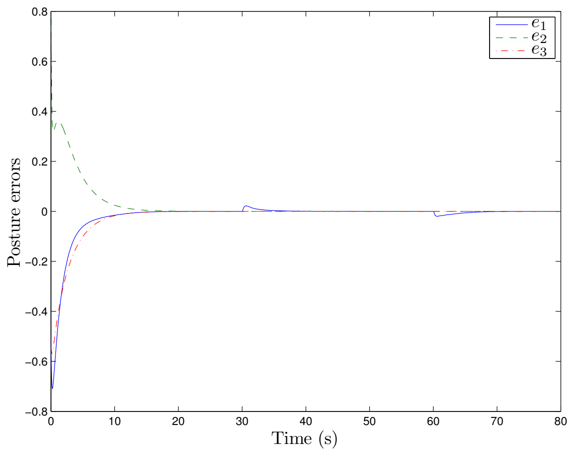

Figure 2: The posture error for the linear reference trajectory.

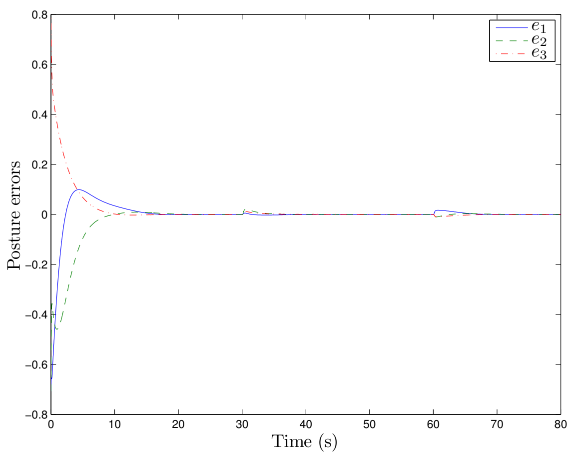

Figure 3: The posture error for the circular reference trajectory.

Figures 4 and 5 show the estimate $\hat{a}$ for the linear and circular references trajectories, respectively. The red dashed line represents the true value of the slip parameter and the blue solid line is the estimated value.

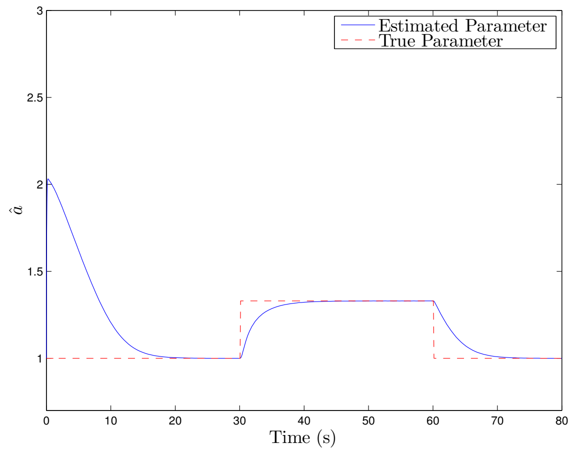

Figure 4: Estimated parameter $\hat{a}$ for the linear reference trajectory.

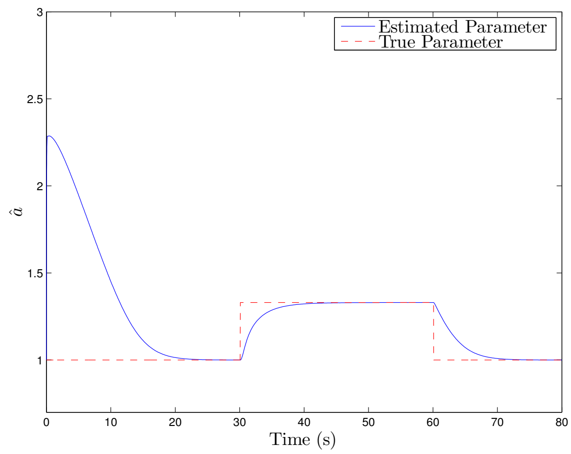

Figure 5: Estimated parameter $\hat{a}$ for the circular reference trajectory.

Figures 6 and 7 show the robot trajectory in the fixed frame F_1 for the linear and circular references trajectories, respectively. The red solid line stands for the reference trajectory, while the blue circle stands for the robot trajectory.

5 Conclusions

This paper provides an adaptive tracking control design for a nonholonomic tracked mobile robot with unknown longitudinal slip. A kinematic model containing the slip parameter is proposed. An update rule is derived to estimate the slip parameter in real time. The proposed adaptive control law ensures that the robot trajectory follows a given reference trajectory. Asymptotic stability of the global closed-loop system is guaranteed

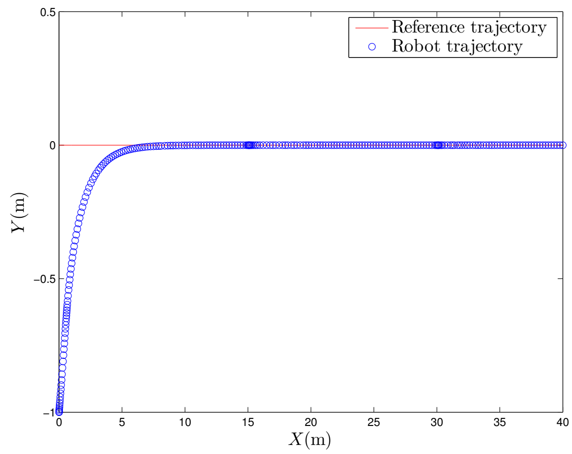

Figure 6: Results for the linear reference trajectory.

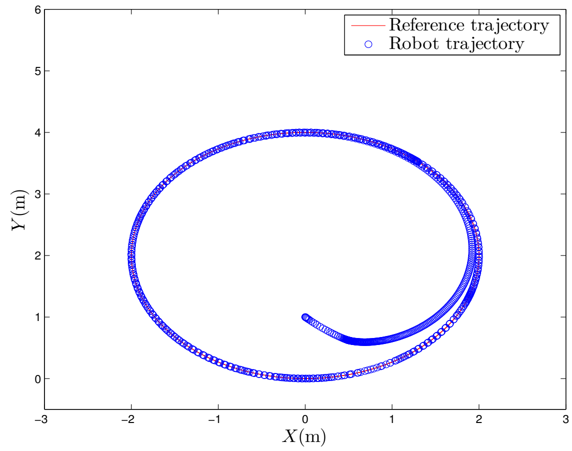

Figure 7: Results for the circular reference trajectory.

using an appropriate Lyapunov function. Numerical results show the effectiveness of the proposed control strategy.

Acknowledgments

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