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A Vibroacoustic Application of Identification and Control for Linear Time-Varying Systems

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Abstract: This paper presents an application of $\mathscr{H}_2$ and $\mathscr{H}_\infty$ gain-scheduled and robust state feedback control for a time-varying vibroacoustic setup whose dynamics is highly sensitive to variation in the ambient temperature. An LPV model is derived for this system using the State-space Model Interpolation of Local Estimates (SMILE) technique. This approach interpolates linear time-invariant models estimated at distinct operating conditions of the system (in this case, different temperatures). The obtained LPV model is used in recently developed gain-scheduled and robust $\mathscr{H}_2$ and $\mathscr{H}_\infty$ synthesis procedures that can consider a priori known bounds on the rate of parameter variation.

Keywords: Gain-scheduled and robust control; LPV identification; $\mathscr{H}_2$ and $\mathscr{H}_\infty$ performance; Linear time-varying systems; Control applications

INTRODUCTION

A main source of noise within aircraft cabins is the vibration of the surrounding structure, usually denoted as structural noise (Berglund et al., 1996; Persson and Björkman, 1988). Research in the area of acoustics has shown that active control strategies are efficient in reducing this noise in the low frequency range, see for example (Sas et al., 1995; Fuller and von Flotow, 1995; Meurers et al., 2002; Kaiser et al., 2003; Alujevic et al., 2008; Donadon et al., 2006). Several of these techniques assume that the plant under consideration is linear time-invariant (LTI), although, for some applications, this is not a realistic assumption. For instance, the structure is frequently subject to temperature changes, and its dynamics may change considerably according to the temperature. To use robust control synthesis techniques for such time-varying systems, the nominal model and the uncertainty bounds should be appropriately determined, due to the important trade-off between performance and robustness. For most practical applications, however, this is a difficult task, and the estimated uncertainty set is in general too conservative. Therefore, a more elaborate strategy should be applied.

To control linear time-varying (LTV) systems, two distinct approaches are commonly used: interpolating gain-scheduling (IGS) control and linear parameter-varying (LPV) control (Packard, 1994; Apkarian and Gahinet, 1995; Shamma and Athans, 1991). In IGS control, the design is split into two parts. First, LTI controllers are designed for linearized models of the system, estimated at several fixed operating conditions. Second, a parameter-dependent controller is obtained by interpolating these LTI controllers. Although the stability of the closed-loop system is not guaranteed, this approach has been successfully used in many industrial applications, e.g., (Nichols et al., 1993; Aouf et al., 2002; De Caigny et al., 2007). In LPV control, on the other hand, stability of the closed-loop system is guaranteed. Several analysis and synthesis techniques for LPV systems have been proposed based on different types of Lyapunov functions. For example, the well-known quadratic stability approach uses a constant Lyapunov matrix, that allows arbitrarily fast variation of the scheduling parameters. Obviously, this yields conservative controllers for practical applications with bounded parameter variation. To mitigate some of the conservatism associated with the quadratic stability-based approaches, many works using parameter-dependent Lyapunov functions have been published. For instance, recently, synthesis procedures for $\mathscr{H}_2$ and $\mathscr{H}_\infty$ gain-scheduled and robust static output feedback controllers have been presented in (De Caigny et al., 2008b,a) for polytopic discrete-time LTV systems with a priori known bounds on the rate of parameter variation.

Most LPV control synthesis techniques need an LPV model of the system that accurately describes the variation of the system dynamics over the workspace. The State-space Model Interpolation of Local Estimates (SMILE) technique, developed in (De Caigny et al., 2008c,d), provides one way to obtain such an LPV model, by interpolating local LTI models estimated at distinct operating conditions of the system.

This paper is organized as follows. First, the vibroacoustic application is introduced. Then, a short outline of the SMILE technique is presented and the technique is used to obtain an LPV model of the vibroacoustic setup. Subsequently, synthesis conditions for gain-scheduled and robust $\mathscr{H}_2$ and $\mathscr{H}_\infty$ state feedback control are given and applied to the obtained LPV model. Concluding remarks follow afterwards.

VIBROACOUSTIC APPLICATION

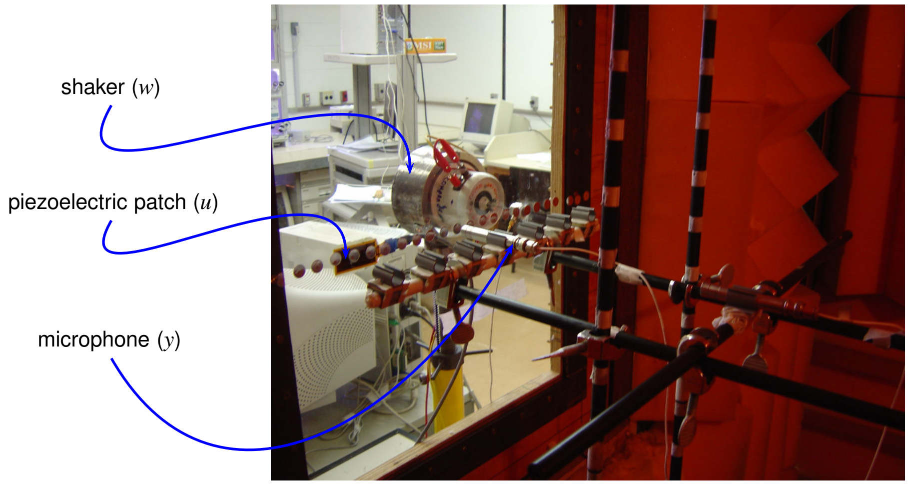

Figure 1: Vibroacoustic setup.

The setup (displayed in Figure 1) consists of a lexan plate, highly sensitive to the ambient temperature, clamped on a rigid baffle, see (Donadon et al., 2006) for details. The exogenous disturbance w that causes the vibration of the plate is provided by a point force driven by a shaker. The control input u, used to attenuate the sound pressure inside a semi-anechoic room, is provided by a flexural moment driven by a piezoelectric patch attached to the plate. The output y is the sound pressure measured by a single microphone, located near the plate. FRFs are measured from the disturbance w and the control input u to output y at four different temperatures $\theta \in \{22.9^\circ, 23.4^\circ, 24.4^\circ, 25.4^\circ\}$. For each operating condition, a 10th-order discrete-time state-space model is estimated. Figure 2 presents the magnitude and phase of the experimental FRFs (black, dashed) and of the estimated models (red, solid) in the frequency range of interest $(120-260Hz)$. The LTI models clearly show a good correspondence to the FRFs. The following section presents the SMILE technique to obtain an LPV model by interpolating these four LTI models.

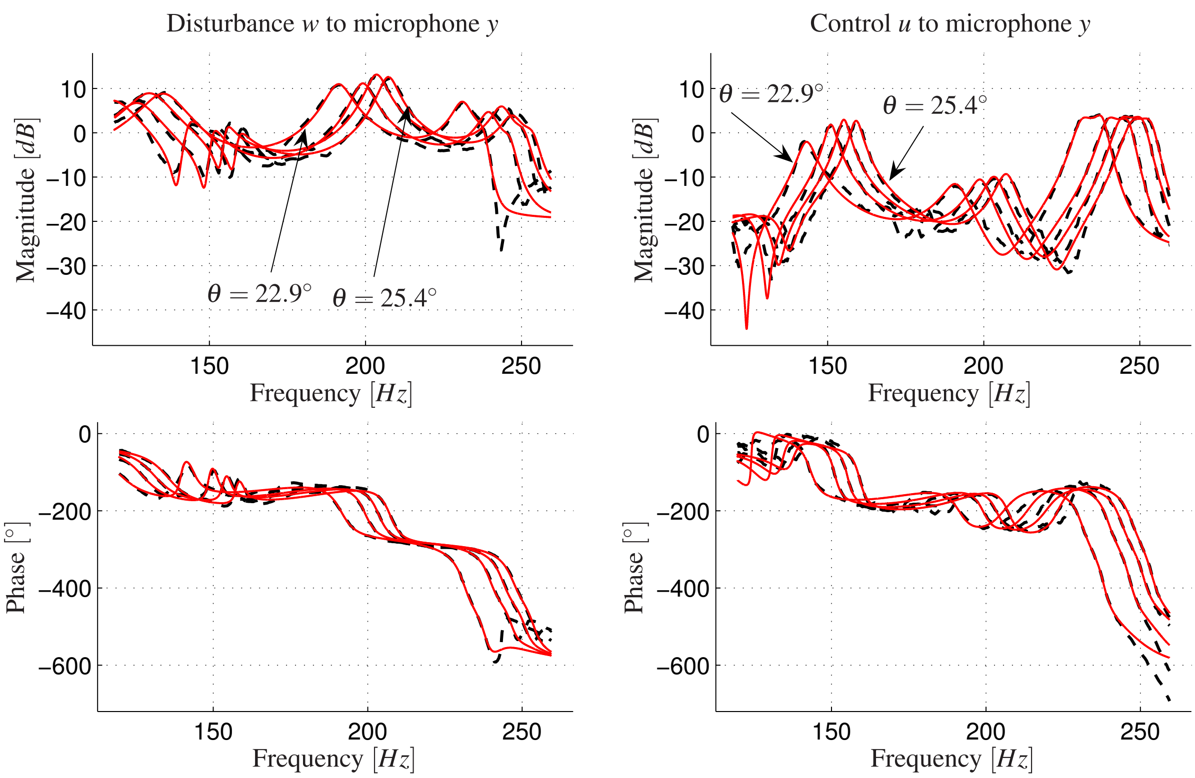

Figure 2: Measured FRFs (black, dashed) and estimated models (red, solid).

SMILE TECHNIQUE

Only a brief outline of the SMILE technique is presented. See (De Caigny et al., 2008c) for details. The notation in this outline is as follows: matrices associated with the interpolating LPV model are denoted using standard math font, e.g., $A_0, \ldots$ Matrices associated with the local models are denoted using San Serif font, e.g., $\mathsf{A}_\ell, \ldots$ The subscript ℓ indicates the index of the local model. Throughout the paper,

$$H := \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right]$$

is used to indicate the discrete-time state-space model

$$H := \begin{cases} x(k+1) = A\,x(k) + B\,u(k), \\ \quad\; y(k) = C\,x(k) + D\,u(k). \end{cases}$$

The interpolating LPV model is chosen to be affine in the scheduling parameter θ and has the following state-space representation:

$$H(\theta) := \left[\begin{array}{c|c} A_0 + A_1\theta & B_0 + B_1\theta \\ \hline C_0 + C_1\theta & D_0 + D_1\theta \end{array}\right], \quad \text{where } A_0, A_1 \in \mathbb{R}^{n\times n},\ B_0, B_1 \in \mathbb{R}^{n\times r},\ C_0, C_1 \in \mathbb{R}^{s\times n} \text{ and } D_0, D_1 \in \mathbb{R}^{s\times r}. \tag{1}$$

Obviously, LPV model (1) can be readily used in the many available LPV control synthesis techniques for affine LPV models. Moreover, when θ is bounded, model (1) can be converted exactly in a polytopic LPV model with two vertices, which is useful since control synthesis for polytopic models has been widely studied.

The aim of the SMILE technique is to estimate the system matrices of model (1) such that the resulting LPV model $H(\theta)$ interpolates the m local LTI models

$$\tilde{\mathsf{H}}_\ell := \left[\begin{array}{c|c} \tilde{\mathsf{A}}_\ell & \tilde{\mathsf{B}}_\ell \\ \hline \tilde{\mathsf{C}}_\ell & \tilde{\mathsf{D}}_\ell \end{array}\right], \quad \ell = 1, \ldots, m, \tag{2}$$

identified at distinct operating conditions θ_ℓ. All local LTI models are assumed to have the same number of states n, the same number of inputs r and the same number of outputs s. As the state-space representation is not unique, the local models (2) cannot be readily interpolated since it is not guaranteed that they are represented with respect to the same state-space basis. Therefore, a similarity transformation matrix T_ℓ needs to be calculated for each local model $\tilde{\mathsf{H}}_\ell$ such that the transformed models

$$\mathsf{H}_\ell := \left[\begin{array}{c|c} \mathsf{A}_\ell & \mathsf{B}_\ell \\ \hline \mathsf{C}_\ell & \mathsf{D}_\ell \end{array}\right] = \left[\begin{array}{c|c} \mathsf{T}_\ell^{-1}\tilde{\mathsf{A}}_\ell\mathsf{T}_\ell & \mathsf{T}_\ell^{-1}\tilde{\mathsf{B}}_\ell \\ \hline \tilde{\mathsf{C}}_\ell\mathsf{T}_\ell & \tilde{\mathsf{D}}_\ell \end{array}\right], \quad \ell = 1, \ldots, m, \tag{3}$$

are defined with respect to the same basis. Once the models (3) have been calculated, an optimization problem can be formulated and solved to find the optimal system matrices of the interpolating LPV model (1).

The SMILE technique consists of 5 steps to compute the interpolating LPV model (see the flowchart in Figure 3), assuming that m MIMO LTI models $\tilde{\mathsf{H}}_\ell$ (1), obtained for fixed operating conditions θ_ℓ, are available. These 5 steps are now briefly presented and applied to the four 10th-order 2-input 1-output LTI models of the vibroacoustic setup.

STEP 1: Choose one input-output combination (i, j) for all original local MIMO models $\tilde{\mathsf{H}}_\ell$ to obtain the local SISO models

$$\tilde{\mathsf{H}}_{\ell,(\mathsf{i},\mathsf{j})} := \left[\begin{array}{c|c} \tilde{\mathsf{A}}_\ell & \tilde{\mathsf{B}}_{\ell,(:,\mathsf{j})} \\ \hline \tilde{\mathsf{C}}_{\ell,(\mathsf{i},:)} & \tilde{\mathsf{D}}_{\ell,(\mathsf{i},\mathsf{j})} \end{array}\right], \quad \ell = 1, \ldots, m.$$

For the vibroacoustic setup, the LTI SISO models $\tilde{\mathsf{H}}_{\ell,(1,2)}$, from the control input u to the microphone y, are chosen.

STEP 2: Calculate the poles p_ℓ and zeros $\mathsf{z}_{\ell,(1,2)}$ of the original local SISO models $\tilde{\mathsf{H}}_{\ell,(1,2)}$. Sort these poles and zeros such that they are in the same order for all local SISO models $\tilde{\mathsf{H}}_{\ell,(1,2)}$. Figure 4 shows the real and imaginary part of the poles and zeros of the four local models as a function of the varying temperature θ. All local models have 5 complex conjugated pole pairs, 1 nonminimum-phase real zero and 4 complex conjugated zero pairs. Based on Figure 4, the poles and zeros of the local SISO models can be easily sorted.

STEP 3: Divide the local SISO models $\tilde{\mathsf{H}}_{\ell,(1,2)}$ into a gain $\mathsf{K}_{\ell,(1,2)}$ multiplied by the series connection of τ_1 1st-order and τ_2 2nd-order submodels $\mathsf{H}^\tau_{\ell,(1,2)}$ and explicitly calculate this series connection to obtain a new state-space representation of the local SISO models (denoted as $\mathsf{H}_{\ell,(1,2)}$). Since all LTI SISO models are 10th-order, they can be

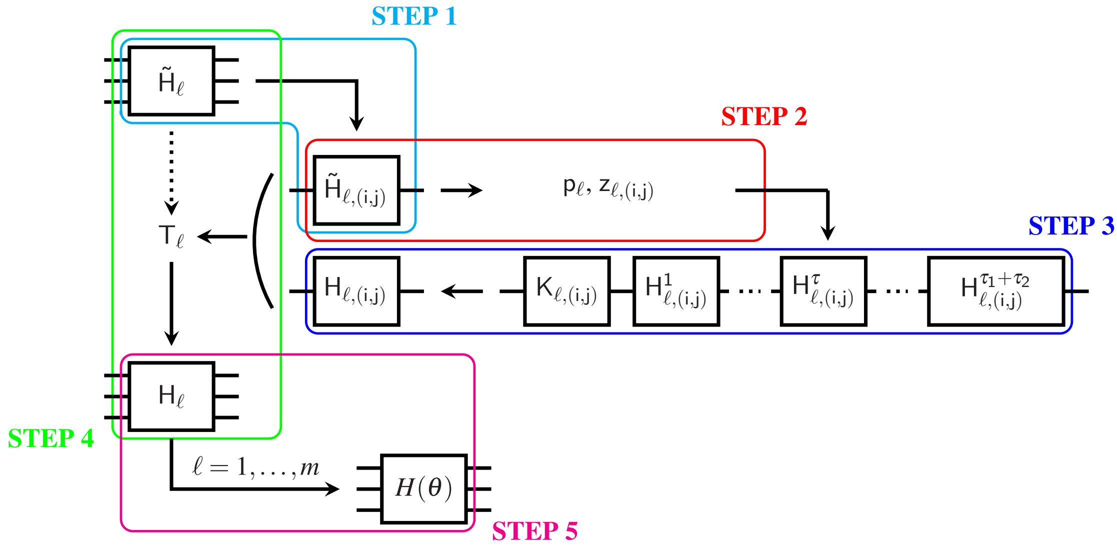

Figure 3: Flowchart of the SMILE technique.

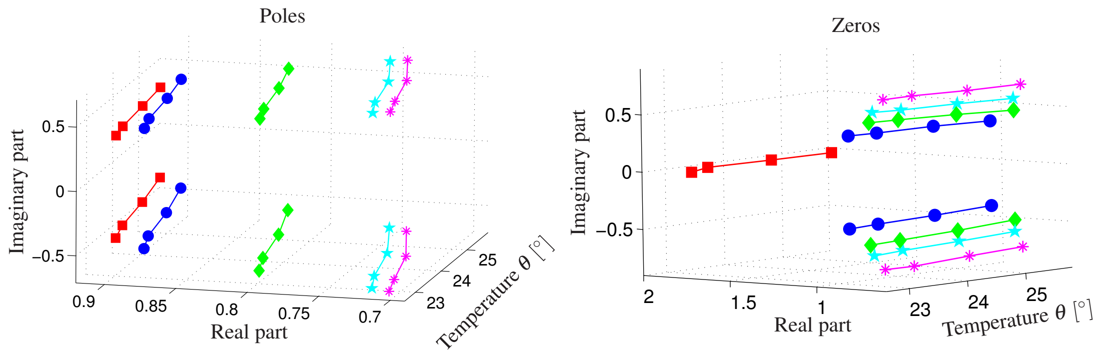

Figure 4: Poles and zeros of the local SISO models $\tilde{\mathsf{H}}_{\ell,(1,2)}$.

represented by a gain multiplied by the series connection of 5 2nd-order submodels. In Figure 4 this division is emphasized by assigning the poles and zeros 5 different markers and colors: poles and zeros with the same marker and color are assigned to the same LTI SISO submodel.

STEP 4: Calculate, for every ℓ, the similarity transformation matrix T_ℓ that transforms the system matrices of $\tilde{\mathsf{H}}_{\ell,(1,2)}$ into those of $\mathsf{H}_{\ell,(1,2)}$. Apply this transformation matrix T_ℓ to the corresponding original MIMO LTI model $\tilde{\mathsf{H}}_\ell$ to obtain the model H_ℓ.

STEP 5: To obtain the system matrices of the interpolating MIMO LPV model (1), the following linear least-squares cost function is minimized

$$E = \sum_{\ell=1}^{m} \|A_0 + A_1\theta - \mathsf{A}_\ell\|_F^2 + \|B_0 + B_1\theta - \mathsf{B}_\ell\|_F^2 + \|C_0 + C_1\theta - \mathsf{C}_\ell\|_F^2 + \|D_0 + D_1\theta - \mathsf{D}_\ell\|_F^2$$

where $\| \ \|_F$ represents the Frobenius norm of a matrix.

Figure 5 compares the 4 local LTI MIMO models (red, solid) to the obtained interpolating LPV model (black, solid with dots), evaluated at 11 equidistantly spaced temperatures in the range $[22.9°, 25.4°]$. The LPV model clearly shows a smooth interpolation of the local MIMO models. This LPV model is now used in the synthesis techniques presented in the next section.

GAIN-SCHEDULED AND ROBUST $\mathscr{H}_2$ AND $\mathscr{H}_\infty$ STATE FEEDBACK

This section presents synthesis procedures for gain-scheduled and robust $\mathscr{H}_2$ and $\mathscr{H}_\infty$ state feedback controllers for discrete-time polytopic LTV systems with known bounds on the rate of parameter variation. First, the modeling of the uncertainty domain is introduced, then the synthesis conditions are given and afterwards state feedback controllers are computed for the LPV model of the vibroacoustic setup obtained in the previous section.

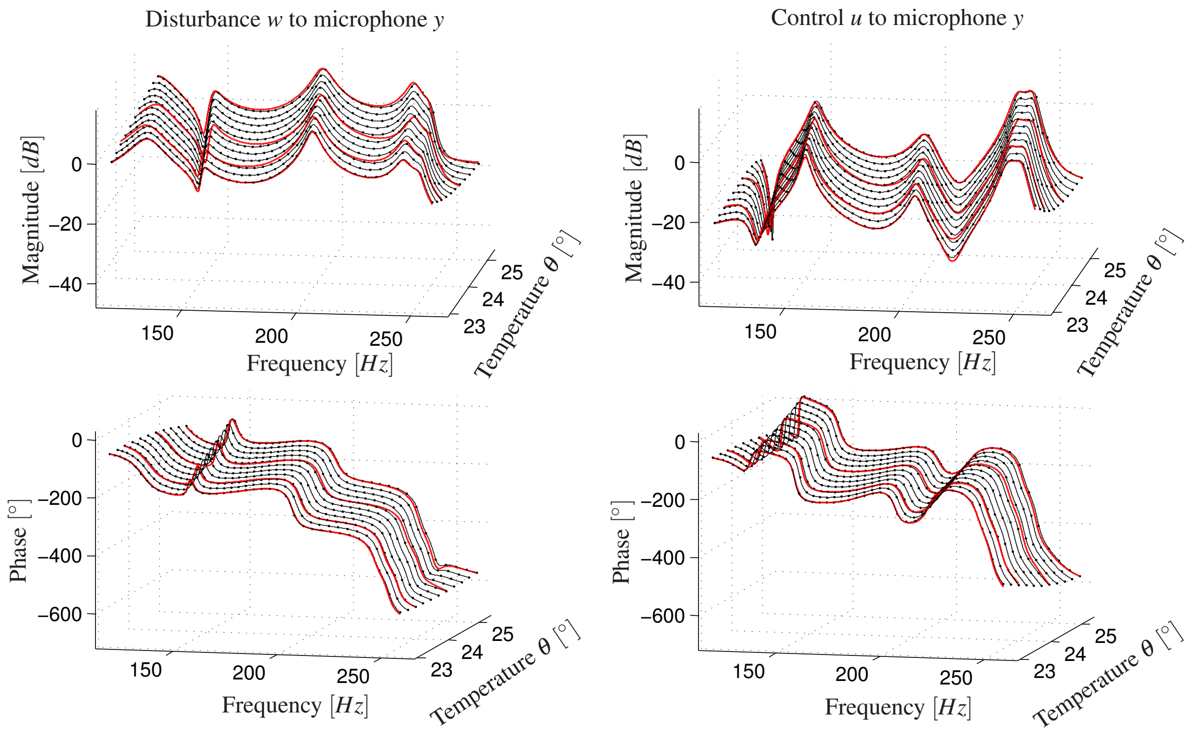

Figure 5: The interpolating LPV model, evaluated at 11 different temperatures (black, solid with dots) compared to the 4 local LTI MIMO models (red, solid).

Modeling of the uncertainty domain

Consider the polytopic discrete-time linear time-varying system

$$H(\alpha) := \left[\begin{array}{c|cc} A(\alpha(k)) & B_w(\alpha(k)) & B_u(\alpha(k)) \\ \hline C_z(\alpha(k)) & D_w(\alpha(k)) & D_u(\alpha(k)) \end{array}\right] \tag{4}$$

where the vector of time-varying parameters $\alpha(k) \in \mathbb{R}^N$ belongs to the unit simplex

$$\Lambda_N = \left\{ \xi \in \mathbb{R}^N : \sum_{i=1}^{N} \xi_i = 1, \xi_i \geq 0, i = 1, \ldots, N \right\},$$

and the system matrices $A(\alpha(k)) \in \mathbb{R}^{n\times n}$, $B_w(\alpha(k)) \in \mathbb{R}^{n\times r}$, $B_u(\alpha(k)) \in \mathbb{R}^{n\times m}$, $C_z(\alpha(k)) \in \mathbb{R}^{s\times n}$, $D_w(\alpha(k)) \in \mathbb{R}^{s\times r}$ and $D_u(\alpha(k)) \in \mathbb{R}^{s\times m}$ belong to the polytope

$$\mathscr{D} = \{ (A, B_w, B_u, C_z, D_w, D_u)(\alpha(k)) : (A, B_w, B_u, C_z, D_w, D_u)(\alpha(k)) = \sum_{i=1}^{N} \alpha_i(k) (A, B_w, B_u, C_z, D_w, D_u)_i, \ \alpha(k) \in \Lambda_N \}.$$

The rate of variation of the parameters $\Delta\alpha_i(k) = \alpha_i(k+1) - \alpha_i(k)$, $i = 1, \ldots, N$, is assumed to be limited by an *a priori* known bound $b \in \mathbb{R}$ such that

$$-b\alpha_i(k) \leq \Delta\alpha_i(k) \leq b(1 - \alpha_i(k)), \quad i = 1, \ldots, N \tag{5}$$

with $b \in [0, 1]$. As recently discussed in (Oliveira and Peres, 2008), less conservative synthesis procedures can be derived by explicitly taking into account that $\Delta\alpha_i(k)$ satisfies (5).

Gain-scheduled control synthesis

The goal is to provide a parameter-dependent state feedback control law $u(k) = K(\alpha(k))x(k)$, with $K(\alpha(k)) \in \mathbb{R}^{m\times n}$, such that the closed-loop system

$$H_{CL}(\alpha) := \left[\begin{array}{c|c} A(\alpha(k)) + B_u(\alpha(k))K(\alpha(k)) & B_w(\alpha(k)) \\ \hline C_z(\alpha(k)) + D_u(\alpha(k))K(\alpha(k)) & D_w(\alpha(k)) \end{array}\right]$$

is exponentially stable with a guaranteed $\mathscr{H}_2$ or $\mathscr{H}_\infty$ performance for all possible parameter variation. A solution to the gain-scheduled $\mathscr{H}_\infty$ state feedback design problem, in terms of a finite set of LMIs, is provided by the next theorem.

Theorem 1 *If there exist, for $i = 1,\dots,N$, matrices $G_i \in \mathbb{R}^{n\times n}$, $Z_i \in \mathbb{R}^{m\times n}$, and symmetric positive-definite matrices $P_i \in \mathbb{R}^{n\times n}$ such that*

$$\begin{bmatrix} (1-b)P_i + bP_\ell & \star & \star & \star \\ G_i^T A_i^T + Z_i^T B_{u,i}^T & G_i + G_i^T - P_i & \star & \star \\ B_{w,i}^T & 0 & I & \star \\ 0 & C_{z,i}G_i + D_{u,i}Z_i & D_{w,i} & \eta I \end{bmatrix} > 0, \tag{6}$$

for $i = 1,\dots,N$ and $\ell = 1,\dots,N$ and

$$\begin{bmatrix} (1-b)(P_i+P_j) + 2bP_\ell & \star & \star & \star \\ G_j^T A_i^T + G_i^T A_j^T + Z_j^T B_{u,i}^T + Z_i^T B_{u,j}^T & G_i + G_i^T + G_j + G_j^T - P_i - P_j & \star & \star \\ B_{w,i}^T + B_{w,j}^T & 0 & 2I & \star \\ 0 & C_{z,i}G_j + C_{z,j}G_i + D_{u,i}Z_j & D_{w,i} + D_{w,j} & 2\eta I \end{bmatrix} > 0,$$

for $\ell = 1,\dots,N$, $i = 1,\dots,N-1$ and $j = i+1,\dots,N$, then the parameter-dependent static state feedback gain

$$K(\alpha(k)) = Z(\alpha(k))G(\alpha(k))^{-1}, \quad \text{with } Z(\alpha(k)) = \sum_{i=1}^{N} \alpha_i(k) Z_i \text{ and } G(\alpha(k)) = \sum_{i=1}^{N} \alpha_i(k) G_i, \tag{7}$$

stabilizes system (4) with a guaranteed $\mathscr{H}_\infty$ performance bounded by $\sqrt{\eta}$.

The proof for Theorem 1 can be found in (De Caigny et al., 2008a). The next theorem provides a finite set of LMIs for the design of a gain-scheduled $\mathscr{H}_2$ state feedback controller for system (4).

Theorem 2 *If there exist, for $i = 1,\dots,N$, matrices $G_i \in \mathbb{R}^{n\times n}$, $Z_i \in \mathbb{R}^{m\times n}$, and symmetric positive-definite matrices $P_i \in \mathbb{R}^{n\times n}$ and $W_i \in \mathbb{R}^{p\times p}$ such that*

$$\begin{bmatrix} (1-b)P_i + bP_\ell & \star & \star \\ G_i^T A_i^T + Z_i^T B_{u,i}^T & G_i + G_i^T - P_i & \star \\ B_{w,i}^T & 0 & I \end{bmatrix} > 0, \tag{8}$$

for $i = 1,\dots,N$ and $\ell = 1,\dots,N$,

$$\begin{bmatrix} (1-b)(P_i+P_j) + 2bP_\ell & \star & \star \\ G_j^T A_i^T + G_i^T A_j^T + Z_j^T B_{u,i}^T + Z_i^T B_{u,j}^T & G_i + G_i^T + G_j + G_j^T - P_i - P_j & \star \\ B_{w,i}^T + B_{w,j}^T & 0 & 2I \end{bmatrix} > 0,$$

for $\ell = 1,\dots,N$, $i = 1,\dots,N-1$ and $j = i+1,\dots,N$,

$$\begin{bmatrix} W_i - D_{w,i}D_{w,i}^T & \star \\ G_i^T C_{z,i}^T + Z_i^T D_{u,i}^T & G_i + G_i^T - P_i \end{bmatrix} > 0, \tag{9}$$

for $i = 1,\dots,N$,

$$\begin{bmatrix} W_i + W_j - D_{w,j}D_{w,i}^T - D_{w,i}D_{w,j}^T & \star \\ G_j^T C_{z,i}^T + G_i^T C_{z,j}^T + Z_j^T D_{u,i}^T + Z_i^T D_{u,j}^T & G_i + G_i^T + G_j + G_j^T - P_i - P_j \end{bmatrix} > 0,$$

for $i = 1,\dots,N-1$ and $j = i+1,\dots,N$, then the parameter-dependent state feedback gain (7) stabilizes system (4) with a guaranteed $\mathscr{H}_2$ performance bounded by ν with ν given by $\nu^2 = \min\limits_{P_i,G_i,Z_i,W_i} \max\limits_i Tr\{W_i\}$.

The proof for Theorem 2 can be found in (De Caigny et al., 2008b).

By combining the LMI conditions presented in Theorem 1 and 2, it is possible to design mixed $\mathscr{H}_2/\mathscr{H}_\infty$ controllers. Multiobjective $\mathscr{H}_2$ and $\mathscr{H}_\infty$ specifications can be imposed on different closed-loop input-output combinations by appropriately selecting the right input-output channels of the open-loop system (4) and applying the control synthesis procedures of Theorem 1 or 2, using the same variables G_i and Z_i for all performance specifications. For each performance specification, however, a different set of Lyapunov matrices can be used. This mixed $\mathscr{H}_2/\mathscr{H}_\infty$ synthesis technique extends the *G shaping paradigm*, presented in (de Oliveira et al., 2002) for uncertain LTI systems, to the class of polytopic LTV systems with bounds on the rate of parameter variation.

Robust control synthesis

Robust state feedback controllers $u(k) = K\ x(k)$ can be easily derived from Theorems 1 and 2, as shown in the following corollaries.

Corollary 1 *If there exist matrices $G \in \mathbb{R}^{n\times n}$, $Z \in \mathbb{R}^{m\times n}$, and symmetric positive-definite matrices $P_i \in \mathbb{R}^{n\times n}$, for $i = 1,\ldots,N$, such that* (6) *holds for $i = 1,\ldots,N$ and $\ell = 1,\ldots,N$, with $G_i = G$ and $Z_i = Z$, for $i = 1,\ldots,N$, then the robust state feedback gain $K = ZG^{-1}$ stabilizes system* (4)*, with a guaranteed $\mathscr{H}_\infty$ performance bounded by $\sqrt{\eta}$.*

Corollary 2 *If there exist matrices $G \in \mathbb{R}^{n\times n}$, $Z \in \mathbb{R}^{m\times n}$, and symmetric positive-definite matrices $P_i \in \mathbb{R}^{n\times n}$ and $W_i \in \mathbb{R}^{p\times p}$, for $i = 1,\ldots,N$, such that* (8) *holds for $i = 1,\ldots,N$ and $\ell = 1,\ldots,N$ and* (9) *holds for $i = 1,\ldots,N$, with $G_i = G$ and $Z_i = Z$, for $i = 1,\ldots,N$, then the robust static output feedback gain $K = ZG^{-1}$ stabilizes system* (4)*, with a guaranteed $\mathscr{H}_2$ performance ν, with ν given by $\nu^2 = \min\limits_{P_i,G,Z,W_i} \max\limits_i Tr\{W_i\}$.*

Obviously, robust multiobjective state feedback controllers can be obtained as well, by combining the results of these two corollaries for different performance specifications.

Control results

The goal of the control design is to minimize an upper bound γ_2 on the closed-loop $\mathscr{H}_2$ performance from the disturbance w to the output z. To obtain realistic controllers, that do not have excessively large control signals, an $\mathscr{H}_\infty$ bound γ_1 is enforced on the control effort. Both gain-scheduled and robust controllers are designed for different bounds on the rate of parameter variation $b \in \{0, 0.5, 1\}$ with the upper bound γ_1 on the closed-loop $\mathscr{H}_\infty$ cost from w to u given by the range $\gamma_1 \in [0.8, 50]$. To use the synthesis conditions of Theorem 1 and 2 and Corollary 1 and 2, the affine LPV model of the vibroacoustic setup, obtained with the SMILE technique, is converted in a polytopic model with two vertices with scheduling parameters

$$\alpha_1 = \frac{\theta - \theta_{min}}{\theta_{max} - \theta_{min}}, \text{ and } \alpha_2 = 1 - \alpha_1, \text{ with } \theta_{min} = 22.9^\circ,\ \theta_{max} = 25.4^\circ.$$

Figure 6 shows the trade-off between the $\mathscr{H}_\infty$ upper bound γ_1 on the control effort and the $\mathscr{H}_2$ upper bound γ_2 on the closed-loop $\mathscr{H}_2$ performance for the different bounds b. Gain-scheduled control designs are indicated with *GS* using dashed lines. Robust control designs are indicated with *R* using solid lines. For each bound $b \in \{0, 0.5, 1\}$, a gain-scheduled and robust $\mathscr{H}_2$ controller without any bound on the control effort was also calculated using the LMIs of Theorem 2 and Corollary 2, thus providing the maximum achievable performance, that is, the minimal achievable value for γ_2 (indicated in Figure 6 with dotted lines for the gain-scheduled case and with dash-dotted lines for the robust case). As expected, the mixed $\mathscr{H}_2/\mathscr{H}_\infty$ control designs always provide worse γ_2 performance compared to the $\mathscr{H}_2$ control design. It is also clear that as the bound b on the rate of variation increases, the performance decreases. For small values of γ_1 the synthesis conditions become infeasible (indicated with squares). As expected, the gain-scheduled controllers outperform the robust controllers.

CONCLUSIONS

This paper presents a vibroacoustic application of identification and control for linear time-varying systems. An LPV model of the system is obtained using the recently developed State-space Model Interpolation of Local Estimates (SMILE) technique, based on the interpolation of state-space LTI models that are estimated for fixed operating conditions of the system. The resulting LPV model is used in mixed $\mathscr{H}_2/\mathscr{H}_\infty$ synthesis techniques for gain-scheduled and robust state feedback controllers. The synthesis procedures explicitly take an a priori known bound on the rate of parameter variation into account, thus reducing the conservatism generally associated with methods that allow arbitrarily fast parameter variation.

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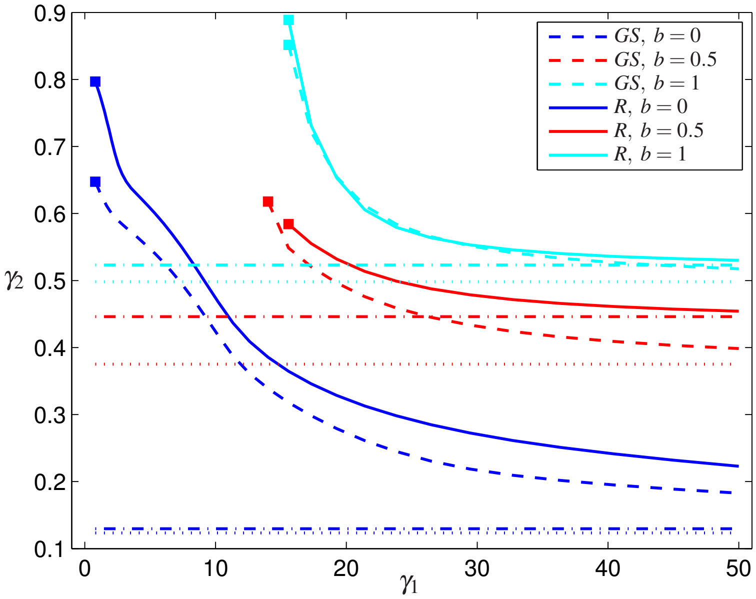

Figure 6: Trade-off between control effort and performance.

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