Abstract—This paper presents preliminary results on the experimental application of a track-following control to an acoustically actuated micro-scanning mirror. Scanning mirrors demand high degree of precision in keeping constant the scanner operational frequency and amplitude of oscillation. This can hardly be achieved by using only open-loop strategies, since these micro devices are very sensitive to environmental conditions, which could cause changes in their natural frequencies. In order to maintain constant the operating frequency and amplitude, a closed-loop control strategy is proposed. The outline of this strategy is that the angular velocity of the scanning mirror must follow a sinusoidal reference signal of predefined frequency and amplitude. This control design is based on the internal model principle. We require that the acoustically actuated micro-scanner must oscillate at 1164Hz with a peak velocity of 0.08 m/sec. The obtained experimental results show that it was possible to control this new kind of optical scanners to the specified frequency and amplitude with small tracking error.

I. INTRODUCTION

Light beam scanners are usually used on laser printers, bar code readers, pre-press color separation systems, laser imagesettings, laser coding, displays, high-resolution inspection systems, laser cutting and marking, confocal microscopes, holographic data storage systems, and instrumentation [14; 16; 18]. These application have in common the need for accurate and repetitive scanning of a laser beam on the working plane, thus, demanding frequent use of closed-loop control strategies [13].

Most of these applications are based on conventional mechanical scanners [19], but a new generation of micro-mechanized miniature devices started to hit the market. There are several different actuation principles for these new devices, such as electrostatic [3], thermal, piezoelectric and electromagnetic [5; 15; 17]. The double-paddle scanners (DPS) are a new class of devices acoustically actuated developed in [8; 9] that can offer advantages over the traditional scanners by operating at high frequencies and by providing large optical deflection angles [2].

The optical deflection angle represents the quality factor $Q$ of the scanner, which is defined as the ratio of the natural frequency to the bandwidth at half power. This is an important parameter that describes the sharpness of the DPS response and the accuracy of the DPS in keeping constant its oscillatory behavior. Generally, the higher the parameter $Q$, the more stable is the DPS, since large $Q$ means that the scanner will vibrate close to its natural resonance frequency.

In general, higher amplitude of excitation results in higher deflection angle, and thus, a higher quality factor. However, there is a limit in the amount of power we can apply to the acoustically actuated DPS system, since this device can easily break due to its micro-dimensions. Moreover, this micro devices are very sensitive to environmental condition, thus, its structural properties can vary significantly, mainly under temperature change and aging effects. Consequently, its natural frequencies changes continually and its performance can degrade significantly under practical applications. For instance, on a typical laser printer application, the pixel position tolerance is about 10% of the pixel width along a scanning line [13].

The scanning mirrors used for industrial applications demand high degree of precision in keeping constant its frequency and amplitude of oscillation. This tight accuracy are hardly to be achieved by only open-loop strategies. Therefore, to guarantee a high quality factor by keeping constant its oscillatory behavior under practical conditions, we propose a track-following control strategy in which a specific point on the surface of the mirror is constrained to follow a predefined sinusoidal reference signal of given frequency and amplitude. This control design is based on the internal model principle [6; 10]. To stabilize the final closed-loop system, we employ a linear quadratic Gaussian (LQG) control problem [4; 11]. There are not yet published experimental results on the closed-loop control for this new class of acoustically actuated DPS device. In this sense, this paper also contributes in presenting preliminary experimental results on controlling this double paddle scanner.

The paper is organized as follows. We present the structural properties of the micro-mechanical double-paddle scanner and its acoustical actuator in sections II and II-A. Next, the natural frequency and the mode shape the scanner must operate are identified via finite element analysis in section II-B. The proposed track-following control strategy is present in section III. Subsequently, we present the experimental results in section IV, and the conclusions in section V.

II. THE DOUBLE-PADDLE SCANNER

The micro-mechanical double-paddle scanner (DPS) used in this work was made of a monolithic single-crystal silicon. It consists of a mirror plate (5mmx3mm) connected via a 2.5mm long bar to two rectangular paddles each of 3.5mmx3.5mm. The whole structure is connected to a frame...
by a 3mm bar at the bottom and by a 3.5mm bar at the top, as shown in Figure 1.

![Fig. 1. The double-paddle scanner.](image)

This type of scanner can be efficiently used as a mechanical oscillator for the deflection of laser beams on laser printers and in bar-code readers [1; 7]. This DPS was designed to be excited by an acoustical device, thus no rotor circuits are built on its surface resulting in a lighter and simpler structure. When acoustic waves, generated by small loud speakers, are directed to the double paddles at a certain frequency, the DPS vibrates in one of its natural shape modes, twisting the mirror around its torsion bar.

A. The Acoustical Actuator

The DPS micro-system was packed inside a polyester resin mount with holes that give access to the two paddles. Figure 2 shows the acoustic actuator. This excitation device consists of two mini loudspeakers mounted in such a way that the generated acoustic waves are guided to the two paddles through two conic-shaped ducts.

![Fig. 2. The acoustic excitation device with the mini-loudspeakers.](image)

The two channels (ducts) are identical and the form of each one was made in such a way that it has a diameter of 26mm at one end, where the loudspeakers were mounted, then decreases to a diameter of 2mm and then increases again to a diameter of 3mm at the outlet, where the DPS is mounted. The form and length of each channel is shown in Figure 3. It should be mentioned that the dimensions given here were suitable and compatible with the dimensions of the mini loudspeakers and the DPS used.

B. Modeling via Finite Element Analysis

In order to determine the operating mode at which the scanner should operate, the identification of the DPS natural frequencies and mode shapes is needed. For this purpose, the DPS was modeled using finite element analysis (FEA). This analysis is based on the form of the operation mode and the displacement magnitudes of the laser scanner (mirror). For the sake of identification of the first four natural modes, a finite element model was developed using ANSYS®.

The numerically calculated mode shapes for the lowest four natural frequencies are shown in Figures 4 and 5. It should be noted that the displacements shown in these figures were normalized for each mode separately. The DPS is designed to operate by reflecting laser beams at the top mirror and thus, the 1st mode of 1237Hz, shown in Figure 4(a), is selected as the operating mode shape. Even though the 4th mode of 4151Hz could also have been used as the operating mode, its corresponding natural frequency is too high for practical applications. The properties of the material used to build the scanner were: mass density \( \rho = 2330 Kg/m^3 \), Young's modulus \( E = 65 GPa \), Poisson ratio \( \eta = 0.09 \), and the bar thickness \( h = 280 \mu m \).
III. PROPOSED CONTROL DESIGN

As described previously, it is desired that the scanner oscillates with a fixed frequency and amplitude. This is equivalent to imposing that either the angular displacement or the velocity of the mirror follows a sinusoidal trajectory. This performance criteria can be efficiently enforced by a track-following control strategy.

Consider the block diagram described in Figure 6 where the proposed control configuration is shown. In this diagram, $P(z)$ is the identified plant and $C(z)$ is the track following controller to be designed. We assume the plant and the controller have respectively the following coprime realization $P(z) = N(z)/D(z)$ and $C(z) = D_c(z)/N_c(z)$. The tracking error is given by $e(k) = r(k) - y(k)$, with the output $y(k)$ defined as the velocity in m/sec of some specific position on the surface of the mirror.

![Fig. 6. Proposed control design.](image)

In this design, the class of reference signals $r(k)$ the controller must follow is described by the transfer functions $P(z)$ and $C(z)$. This is equivalent to imposing that $e(k)$ tend to zero as $k \to \infty$, and the following state-space representation

$$x(k+1) = Ax(k) + Bu(k)$$

$$y(k) = Cx(k)$$

with some unknown initial condition $x_0$. Note that the minimal polynomial of $A$ is $D_c(z)$.

To solve this track-following control problem, we shall adopt a classical approach that incorporates a model of reference into the feedback loop [6]. The idea basically consists of two steps:

1. We let $\phi(z)$ contain the unstable poles of the reference model $R(z)$, then we introduce the transfer function $\phi(z)^{-1}$ inside the loop as shown in Figure 7.

2. We design a compensator $C(z)$ such that the feedback system is asymptotically stable.

This procedure of duplicating the dynamic of the reference signal inside the loop is known as the internal model principle.

![Fig. 7. Asymptotic tracking with an internal model.](image)

It can be shown that this design is robust to parameter perturbations in the controller and in the plant so long as all roots of

$$D_c(z)D_p(z)\phi(z) + N_c(z)N_p(z)$$

remain to have negative real parts. To see this, note that the transfer function for the tracking error $e(z) = r(z) - y(z)$ can be written as

$$e(z) = \frac{D_cD_pN_c\phi}{D_cD_p\phi + N_cN_pD_p}$$

Since all the unstable roots of $D_c(z)$ are canceled by $\phi(z)$, all poles of $e(z)$ have negative real parts. Hence we have $e(k) \to 0$ as $k \to \infty$.

We assume in this design that no zero-pole cancellations occurs between the reference and the plant, otherwise we are not able to asymptotically track the reference signal. With this assumption, the augmented system $P(z)\phi(z)^{-1}$ is also stabilizable and detectable. Let this augmented system has the following state-space representation

$$x(k+1) = Ax(k) + Bu(k)$$

$$y(k) = Cx(k)$$

It now remains to design a dynamic stabilizing controller $C(z)$ for the augmented system (1). The input for this controller is the tracking error $e(k)$. For this purpose, we address the design as an linear quadratic Gaussian (LQG) control problem [4], which by the separation principle, can be split into an optimal linear quadratic regulator (LQR) problem and an optimal Kalman filtering problem.

The stabilizing state feedback control law $u(k) = Kx(k)$ for the LQR problem is obtained from the solution of the Riccati equation

$$X = A^T(X - XB(B^TAXB + R)^{-1}B^TX)A + Q$$

(2)

associated to the following cost function:

$$J(x,u) = \frac{1}{N} \sum_{k=0}^{N} x^T(k)Qx(k) + u(k)Ru(k)$$

where $Q = Q^T$ and $R = R^T$ are the respective weights on for the variable $X$ the state $x(k)$ and the control input $u(k)$. After solving (2), the optimal gain $K$ is given by $K = (R + B^TAXB)^{-1}B^TXA$.

However, we do not have the states available for feedback. Thus, we shall also construct an asymptotic observer driven by the measured tracking error $e(k)$. This estimator is given by

$$\tilde{x}(k+1) = (A + LC)\tilde{x}(k) + Bu(k) + Le(k)$$

where the gain $L$ is chosen such that the matrix $A + LC$ is stable. Now, defining the control law as $u(k) = K\tilde{x}$, the dynamic output feedback controller $C(z)$ is given by

$$\tilde{x}(k+1) = (A + BK + LC)\tilde{x}(k) + Le(k)$$

$$u(k) = K\tilde{x}$$

(3)

The gain $L$ in our design is obtained as the solution of a Kalman filtering problem. For this purpose, we need to take into account the process noise $w(k)$ and the measurement
noise $v(k)$. These are uncorrelated white noise with zero mean and covariances respectively given by

$$\varepsilon \{ w(k)w(k)^T \} = M, \quad \varepsilon \{ v(k)v(k)^T \} = N.$$  

The Riccati equation associated with the Kalman filter has the same form as (2). Therefore, we can simply apply the change of variables $A \rightarrow A^T$, $B \rightarrow C^T$, $Q \rightarrow M$, $R \rightarrow N$, and directly use (2).

IV. EXPERIMENTAL RESULTS

To apply the proposed control strategy presented in the previous section III, one still needs to validate the natural frequencies and mode shapes obtained in section II-B by performing an experimental modal analysis on the scanner, and to estimate a discrete-time model using the measured experimental FRF data from this experimental modal analysis.

A. Experimental Modal Analysis

The acoustical actuator described earlier was used for the excitation of the DPS. A random signal used as the input reference signal in the range of 1 Hz to 6 kHz was sent to the loudspeakers. The two paddles were simultaneously fed with this acoustical random signal. The out-of-plane velocity of the DPS surface was measured with a laser Doppler vibrometer (Polytec OFV 330) aiming the laser beam orthogonal at its surface. The excitation and response signals were acquired using the HP-VXI data acquisition system and the LMS CADA-X software package. The Measurements were taken at 30 locations on the surface of the DPS, as shown in Figure 8. In order to prevent the influence of airborne sound on the DPS vibration, it was necessary to reduce the outside noise. This was done by measuring the frequency response functions (FRF) in a semi-anechoic room.

![Figure 8. The fabricated DPS (a) with the grid of points measured on its surface (b).](image)

The natural frequencies of the DPS were calculated from the measured FRFs using the LMS® modal analysis package. As the DPS is designed to operate by reflecting laser beams incident on its mirror, only the 1st and the 4th natural frequencies are of specific concern in this investigation. The experimentally identified natural frequencies of the 1st and 4th modes are, respectively, 1165Hz and 3971Hz. These values are in accordance with the natural frequencies numerically obtained using the FEA model in section II-B. We do not have space here to show all the measured FRFs, so we only present in Figure 9 the FRF measured at the position 20 on the surface of the mirror, since this is the location we intend to control.

![Figure 9. FRF measured on the DPS surface at location 20.](image)

We now need to estimate a discrete-time model using this FRF. Since, we will control only the first mode of the scanner, which corresponds to the natural frequency of 1164HZ, we do not need to estimate a model in the entire frequency, thus, we restrict the frequency range to 950 – 1500Hz.

B. System Identification

The discrete-time transfer function from the control input $u(k)$ to the measurement output $y(k)$ can be represented by the auto-regressive with exogenous input (ARX) model [12] given by

$$D_p(z)y(k) = N_p(z)u(k) + e(k).$$

(4)

This model was identified by a least-square parameter estimation using the experimental FRF data corresponding to the position 20 on the surface of the mirror (Figure 9).

![Figure 10. Estimated model.](image)
The estimated 4th order discrete-time ARX model is presented in Figure 10. The order \( N = 4 \) was the smallest one that still provided a good fit. The numerical values of the polynomials \( D_p(z) \) and \( N_p(z) \) are respectively given by

\[
D_p(z) = 1 - 3.65488z^{-1} + 5.26305z^{-2} - 3.51507z^{-3} + 0.925139z^{-4}
\]

and

\[
N_p(z) = -0.000065z^{-1} + 0.000176z^{-2} - 0.000168z^{-3} + 0.000054z^{-4}
\]

For these numerical values, we can apply the proposed control strategy presented in section III.

C. Applying the Proposed Control Design

Following our control strategy, we shall impose that the velocity of the position 20 on the surface of the mirror (Figure 8) must track a given sinusoidal reference. The choice of velocity rather than displacement is due to the fact that the laser vibrometer measures by default velocity. Nevertheless, this is equivalent to imposing a sinusoidal displacement.

The reference signal the scanner must asymptotically track is given by \( r(t) = X \sin(\omega t) \), with \( X = 0.08 \) m/sec and \( \omega = 2\pi \times 1164 = 7313.6 \) rad/sec. Considering that the sampling time is \( T_s = 5 \times 10^{-5} \) sec, the reference \( R(z) \) has the following discrete-time transfer function representation:

\[
R(z) = \frac{N_r(z)}{D_r(z)} = 0.08 \frac{\sin(\omega T_s)z}{z^2 - 2\cos(\omega T_s)z + 1}.
\]

For this reference, we let the transfer function \( \phi(z) \) contain its poles, i.e., we let \( \phi(z) = z^2 - 2\cos(\omega T_s)z + 1 \). Using this \( \phi \) and the plant equation \( P(z) = N_p(z)/D_p(z) \) given in (4), the augmented system can be readily written in the form (1). It remains now to chose the weighting matrices for the LQG problem. The weighting matrices \( Q \) and \( R \) were set to \( Q = I \) and \( R = 10^{11}I \). These values were obtained by increasing \( R \) until the control input \( u(k) \) was sufficiently small. A limiting voltage of 2 volts has been adopted as the maximum allowed potential difference that can be applied to the speakers. For the Kalman filter design we set process and noise covariances to be respectively given by \( M = 10^{-5}I \) and \( N = 25 \times 10^{-3} \).

The control gain \( K \) and the observer gain \( L \) are obtained by solving the Riccati equation (2) respectively with the pair of weights \((Q,R)\) and \((M,N)\). These values are respectively given by

\[
K = \begin{bmatrix}
-2.1275 \times 10^{-08} & -1.2200 \times 10^{-08} & 1.4578 \times 10^{-08} \\
... & 5.3218 \times 10^{-08} & -7.7044 \times 10^{-04} & 5.6274 \times 10^{-03}
\end{bmatrix}
\]

and

\[
L = \begin{bmatrix}
-3.1308 \times 10^{-01} & 7.7399 & 1.0653 \times 10^{-01} \\
... & -1.2319 & 4.5898 \times 10^{-01} & 2.8032 \times 10^{-01}
\end{bmatrix}^T.
\]

With these gain, the final compensator is the series connection between the controller (3) and the transfer function \( \phi(z)^{-1} \).

D. Simulation Results

The configuration for the experimental setup is shown in Figure 11. This makes possible the fast prototyping of the controller. The fourth order controller has been implemented into a DS1103 PPC controller board from the dSPACE expansion box using the simulink diagram presented in Figure 12.

The output of the DSP board goes directly to an analog low-pass filter with a cut-off frequency of 3KHz. This is necessary in order to smooth the command issued to the acoustic actuator, the mini-loudspeakers. The out-of-plane velocity of position 20 at the DPS surface (Figure 8) is measured with a laser Doppler vibrometer as shown in Figure 11.

The controller calculated in the previous section was implemented in the dSPACE system as described in the simulink diagram given in Figure 12. The gain of 0.25 denoted by “laser” in this figure corresponds to the sensitivity of the vibrometer, which was set to 0.025 \( \mu m/s/v \) (micrometer per volts) multiplied by a factor of 10. This factor of 10 and the output gain 1/10 shown in Figure 12 are due to the scalar factors of the AD and DA converter of the dSPACE board.

The sampling frequency has been set to 20KHz, which correspond to a sampling period of \( T_s = 5 \times 10^{-5} \). To prevent against an excessive control input over the acoustic actuator that could break the micro-scanner, a block saturation was included in the loop with a limit of \( \pm 2 \) volts.
The results obtained from the experiment running in real-time are present in Figure 13. One sees that the output of the system $y(k)$, corresponding to the position 20 on the surface of the mirror (Figure 8), was able to follow the given reference signal $r(k)$. After 30 seconds, the standard deviation of the tracking error $e(k) = r(k) - y(k)$ was about 3.486 × 10^{-4} and the peak value was below 1.8 × 10^{-3}. This error is mainly due to round-off error and aliasing, since we are not sampling at a frequency multiple of 1164Hz, thus can be yet minimized [11].

V. CONCLUSIONS

Preliminary results on the experimental application of a track-following controller are presented. The methodology is applied to an acoustically excited double-paddle laser scanning mirror, that can be used in laser printer, bar-code readers, confocal microscopes, among other applications. From the industrial point of view, the scanner is required to operate monotonically with respect to its optical angle, frequency and oscillation amplitude. These parameters could suffer variation due to aging effects and temperature changes. To maintain high operating precision, a closed loop control strategy is applied using the measured tracking error as input. To design this controller, the natural frequencies and mode shapes were identified via an experimental modal analysis.

VI. ACKNOWLEDGMENTS

The authors would like to thank Lazaro Donadon for his valuable help with the experimental setup.

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