Observer-based state-feedback versus $H_\infty$ output feedback control solved by LMI approach for applications in smart structures

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Abstract
This paper aims with the use of linear matrix inequalities approach (LMIs) for application in active vibration control problems in smart structures. A robust controller for active damping in a panel was designed with piezoelectrical actuators in optimal locations for illustration of the main proposal. It was considered, in the simulations of the closed-loop, a model identified by eigensystem realization algorithm (ERA) and reduced by modal decomposition. We tested two different techniques to solve the problem. The first one uses LMI approach by state-feedback based in an observer design, considering several simultaneous constraints as: a decay rate, limited input on the actuators, bounded output peak (output energy) and robustness to parametric uncertainties. The results demonstrated the vibration attenuation in the structure by controlling only the first modes and the increased damping in the bandwidth of interest. However, it is possible to occur spillover effects, because the design has not been done considering the dynamic uncertainties related with high frequencies modes. In this sense, the second technique uses the classical $H_\infty$ output feedback control, also solved by LMI approach, considering robustness to residual dynamic to overcome the problem found in the first test. The results are compared and discussed. The responses shown the robust performance of the system and the good reduction of the vibration level, without increase mass.

1 Introduction

Vibration attenuation is an important goal in many engineering applications, particularly in aerospace industry. Active vibration control (AVC) in distributed structures is of practical interest because of the demanding requirement for guaranteed stability. This is of particular importance in light structures as the generally have low degree of internal damping [1].

There are many robust control techniques in literature that outline these problems. In this research work we have chosen a recent technique involving linear matrix inequalities (LMI) due its advantages when compared to conventional techniques. LMI contributes to overcome many difficulties in control design. In the last decade, LMI has been used to solve many problems that until then was unfeasible through others methodologies, due mainly by powerful algorithms to solve convex optimization problem, as for instance, the interior point method, [2] and [3].

The main purpose of the present work is to apply two different methods for vibration control solved through popular LMI framework. The first one is a state-feedback synthesis via LMI considering parametric uncertainties described by politopic system. This procedure was firstly proposed by [4] and it is discussed in great details in [2]. In this direction of solution, not every states usually are available as measurements, so it was implemented a dynamic observer, also, using LMI techniques to assure adequate performance in the estimation procedure. In the other hand, the second way discussed in this research work uses output feedback technique, then it is not need to design an estimator. Besides that, the problem was solved by using $H_\infty$ approach, thus we can use the power of small gain theorem to work with dynamic
uncertainties relative to residual modes. So, in this way, if there is a factible solution satisfying the requirements specified, it will be assured the absence of spillover effects. The solution method used to solve the $H_\infty$ problem it based on LMI technique proposed in [5]. This procedure is already implemented in the LMI toolbox of the Matlab®, [3].

This paper includes a discussion of the identification system procedure using eigensystem realization algorithm and the state-space realization in modal coordinates. In the sequence, it is presented the LMIs controllers and the respective adopted methodologies. For illustration purpose, the paper concludes with an example in a panel structure with two pairs of coupled piezoelectric actuators. The output signal for feedback obtained by accelerometer. Finally, in the conclusions is suggested some possible future directions of research.

2 Identification system

2.1 State-space realization in modal coordinates

The modal state-space is a standard used by structural control engineering, and the linear model is described by the following equations:

$$\dot{x} = Ax + B_1w + B_2u, \quad y = Cx + D_1w + D_2u$$

where $A$ is the dynamic matrix, $B_1$ is the matrix of disturbance input, $B_2$ is the matrix of control input, $C$ is the output matrix, $w$ is the vector of disturbance input, $u$ is the vector of control input, and $y$ is the output vector. The matrices $D_1$ and $D_2$ are the direct-transmission terms and are usually used when acceleration measurements are available, [6]. The state vector $x$ of the modal coordinates consists of $n$ independent components, $x_i$, that represent a state of each mode. The $x_i$ (ith state component) can be given by:

$$x_i = \left\{ \begin{array}{c} q_i \\ q_i \end{array} \right\}, \quad \text{where } q_{mi} = \zeta_i q_i + \frac{\dot{q}_i}{\omega_i}$$

where $q_i$ is the ith modal displacement, $\dot{q}_i$ is the ith modal velocity, $\zeta_i$ is the ith modal damping, $\omega_i$ is the ith natural frequency and $n$ is the number of modeled modes. The modal state-space realization is characterized by the block-diagonal dynamic matrix and the related input and output matrices:

$$A = \text{diag}(A_{mi}) \quad B = \begin{bmatrix} B_{m1} \\ B_{m2} \\ \vdots \\ B_{mn} \end{bmatrix}, \quad C = \begin{bmatrix} C_{mi} & C_{m2} & \cdots & C_{mn} \end{bmatrix} \quad \text{with } A_{mi} = \begin{bmatrix} -\zeta_i \omega_i & \omega_i \\ -\omega_i & -\zeta_i \omega_i \end{bmatrix}$$

where the subscript $(.)_{mi}$ is relative to ith dynamic mode and $A_{mi}$ is one possible form to block-diagonal dynamic matrix. More information about this topic can be found in [7].

The obtaining of a low order model is fundamental for a controller successful, so a reduced-order model can be obtained by truncating the states considering the canonical modal decomposition. From the Jordan canonical form, it can be obtained:

$$\begin{bmatrix} \dot{x}_c \\ \dot{x}_r \end{bmatrix} = \begin{bmatrix} A_c & 0 \\ 0 & A_r \end{bmatrix} \begin{bmatrix} x_c \\ x_r \end{bmatrix} + \begin{bmatrix} B_{1c} \\ B_{1r} \end{bmatrix} w + \begin{bmatrix} B_{2c} \\ B_{2r} \end{bmatrix} u$$

$$y = \begin{bmatrix} C_c & C_r \end{bmatrix} \begin{bmatrix} x_c \\ x_r \end{bmatrix} + D_1 w + D_2 u$$

(4)
where the subscripts \( (.)_c \) and \( (.)_r \) mean controlled modes (low frequencies) and residual modes (high frequencies), respectively.

### 2.2 Identification results

For analysis purposes, a finite dimensional system was obtained experimentally in a bandwidth of interest. It was considered an aluminum plate structure with dimension of 0.2 x 0.2 x 0.002 m of length, width, and thickness, respectively. The properties of aluminum are: Young’s modulus = 70 GPa and density = 2710 kg.m\(^{-3}\). The number of electrical dof changes as a function of the number of piezoelectric elements considered (2 dof by PZT), [8]. The properties of PZT, with 0.02 x 0.02 x 0.00027 m of length, width, and thickness, respectively, are based on material designation PSI-5A-S4 (Piezo Systems® Inc.). Figure 1 shows the measurement setup.

![Figure 1: (a) Schematic diagram of the measurement setup. (b) Picture of the measurement setup.](image)

The optimal positions of the actuators were chosen using system norm \( H_\infty \) as objective function, solved by LMIs frameworks. The analytical model was obtained by FEM considering the electromechanical coupling between PZTs and the host structure. This technique of optimal location could be implemented using other indices, for instance, \( H_2 \) or Hankel norm. However, it is not discussed here for clearness of the present proposal. More details can be found in [9] and [7].

The experimental tests were performed by exciting with two different inputs (one by impact hammer, disturbance input, and another one by a pair of PZT actuator, control input). The output (signal for feedback) was obtained in a single point, so, one have two frequency response functions (FRF). The output signal was measured with an accelerometer, model 352A10 PCB Piezotronics®. The impact hammer used was the model 086C04 from PCB Piezotronics®. In this experiment the software SignalCalc ACE® is used to generate a swept sine signal (bandwidth 0-1000 Hz) for the PZT, and, also to realize the data acquisition. A ten-mode model was identified using eigensystem realization algorithm (ERA), [6]. In fact, the real system has nine modes, but ERA adds a highly damped computation mode to improve the identification. The measurements are sampled at 1.25 Hz in the bandwidth of 0-1000 Hz.

The FRFs of real system and of the identified model are shown in fig. 2, for excitation with an impact hammer and for excitation with PZT actuators, respectively. The transfer functions \( G_1 \) and \( G_2 \) are relative with the state-space realization \((A, B_1, C, D_1)\) and \((A, B_2, C, D_2)\), respectively. For design, a fourth order model is obtained by truncating the model. The FRF magnitude plot \( G_1 \) and \( G_2 \) for reduced and residual model are shown in fig. 3.
3 LMI controller

3.1 Controller 1: observer-based state-feedback control

The problem to be investigated is the state-feedback control, with the linear control law, given by:

$$\mathbf{u} = \mathbf{K}_c \mathbf{x}_c$$  \hspace{1cm} (5)

where $\mathbf{K}_c$, the state-feedback gain, must be found. In this formulation, it was ignored the residual modes in the controller design. Considering a polytopic system:
\[ \dot{x}_c = A_c(t)x_c + B_{1c}w + B_{2c}u \]
\[ y_c = C_c x_c + D_{1c}w + D_{2c}u \]

where \( A_c(t) \) is described by a list of vertexes in a convex space \( \text{Co} \) and \( \text{nv} \) is the number of vertexes. The closed-loop system of eq. (6) is quadratically stabilizable (via linear state-feedback) if and only if the following LMI are feasible:

\[ Q > 0, \quad 2\alpha Q + A_{ci}Q + QA^T_{ci} + B_{2c}Y + Y^TB^T_{2c} < 0 \]

where \( A_{ci} \) is the \( i \)th vertex of the polytopic system, \( i=1,...,\text{nv} \), \( Q \) is a symmetric, positive and defined matrix, and \( Y=GQ \), [2]. The decay rate \( \alpha \) are imposed during the closed-loop design.

For a given feedback gain \( K_c \), the output energy of the system from eq. (6) is upper bounded (bounded output peak) by \( x_c(0)^TQ^{-1}x_c(0) \) if \( Q \) also satisfies the LMI below:

\[ Q > 0, \begin{bmatrix} Q & x \\ x^T & \beta \end{bmatrix} > 0 \]
\[ \begin{bmatrix} A_{ci}Q + QA^T_{ci} + B_{2c}Y + Y^TB^T_{2c} \\ C_c Q + D_{2c}Y \end{bmatrix} \leq 0 \]

where \( I \) is the identity matrix. Regarding \( Y \) as a variable, one can find a state-feedback gain \( K_c \) that guarantees output energy less than \( \beta \) by solving the LMI problem given by eq. (8).

When the initial condition, \( x_c(0) \), is known, it is also possible to find an upper bound on the norm of the control input. Therefore, the constraints \( ||u|| \leq \mu \) is enforced at all times \( t>0 \) if the LMI below hold, [10]:

\[ \begin{bmatrix} 1 \\ x(0) \\ Q \end{bmatrix} \geq 0, \begin{bmatrix} Q & Y^T \\ Y & \mu^2I \end{bmatrix} > 0 \]

where \( \mu \) is the maximum value of the amplitude of the control input. The problem above can be solved using interior-point methods, [3]. For each initial condition, the input \( u \) and the output \( y \) assure:

\[ \forall t \geq 0, \begin{cases} ||u|| \leq \mu e^{-\alpha t} \\ ||y|| \leq \beta e^{-\alpha t} \end{cases} \]

The optimal feedback gain is given by:

\[ K_c = YQ^{-1} \]

where \( Y \) and \( Q \) are optimal solution from LMIs problems given by eq. (7), (8) and (9) simultaneously with \( \mu, \beta \) and \( \alpha \) known. For more details see [2] or [10].

In this work is considered the design of a deterministic observer to estimate the modal states not available. So, the input control is:

\[ u = K_c \bar{x}_c \]

where \( \bar{x}_c \) is the estimated controlled modal state vector. One can write the linear equation of modal observer in the form:
where $L$ is the observer gain matrix, which can be obtained by different techniques. In this work we, also, use LMI. It is possible to find an observer gain through the solution of the following LMI, [2]:

$$P > 0 \quad 2\gamma P + A_i^T P + PA_i + W + C_i^T W^T < 0$$  \hspace{1cm} (14)$$

where $A_i$ is $i$th vertex polytopic system, $i=1,...,nv$ and $\gamma$ is the decay rate of the observer, with $\gamma >> \alpha$. To every $P$ and $W$ satisfying these LMI, there corresponds a stabilizing observer. The observer gain is given by:

$$L = P^{-1}W$$  \hspace{1cm} (15)$$

where $P$ and $W$ are solution from LMI problem given by eq. (14).

This optional way of solution has two problems. The first one is the effect of the input control in the residual modes. It can hinder the performance of the closed-loop system, although can not drive the system for unstability. This effect is known as control spillover. The second one is more dangerous and it can turn unstable the system. It is known as observation spillover. Observation spillover is due to contribution of the residual modes in the output signal used to feedback the system. In order to overcome these effects, one can use many strategies, some of them are well-known in the structural control literature, [11]. The next option of controller presented in the paper uses $H_\infty$ theory to warranty the absence of the spillover effects in the system.

### 3.2 Numerical application for controller 1

One of the biggest advantage of LMI approach is the possibility of to deal with constraints and uncertainties simultaneously. This example considers a possible variation of $\pm 15\%$ in the first and second natural frequencies. So, we have two uncertainty parameters:

$$\omega_{n1} \in [\omega_{n1}^{\text{min}} = 0.85 \omega_{n1}, \omega_{n1}^{\text{max}} = 1.15 \omega_{n1}]$$  \hspace{1cm} (16)$$

$$\omega_{n2} \in [\omega_{n2}^{\text{min}} = 0.85 \omega_{n2}, \omega_{n2}^{\text{max}} = 1.15 \omega_{n2}]$$  \hspace{1cm} (17)$$

![Figure 4. Plant families of the system with all four vertexes.](image-url)
Considering these two uncertainties parameters, one have 4 vertexes of a polytopic system (V1, V2, V3, and V4). The FRF plot $G_1$ and $G_2$ for reduced model are shown in fig. 4 for all four open-loop vertexes.

![Figure 5. FRF for uncontrolled and controlled system in the nominal condition.](image)

The LMI regulator (controller + observer) was designed considering all vertexes of the system simultaneously. In this case, it was obtained a robust controller that mathematically guarantees the specifications inside of the convex space. The regulator is obtained from the solution of LMI problem of equations (7), (8), (9) and (14), with: $x_c(0) = [-0.01 0 -0.01 0]^T$, $\mu = 10$, $\beta = 2$, $\alpha = 5$ and $\gamma = 3\alpha$.

![Figure 6. Time domain response for open-loop and closed-loop for the first four modes, considering the system in the nominal condition.](image)

Figure 5 compares the FRF magnitude plots of the uncontrolled and controlled system for the $G_1$ transfer function. As a result of the active damping, the resonance peaks of the controlled modes are reduced. Furthermore, the amplitudes of some of the other modes, which are not explicitly included in the
controller, are also reduced, as for instance the third and sixth modes were attenuated 3.4 dB and 7.0 dB, respectively. Nevertheless, some peaks increase the magnitude, as for instance the fourth and eighth modes were increased the magnitude in 9.0 dB and 4.3 dB, respectively. The amplitude of some peaks can increase because the controller leads to control spillover.

![Figure 7. Time domain physical response for uncontrolled and controlled system.](image)

Figures 6 show the response in time domain for uncontrolled and controlled system and some residual modes, considering -0.01 m of modal displacement as initial condition in the two firsts modes. Only the figure 6a shows the response for uncontrolled system. Clearly, one observes a low influence of the high frequency dynamics in the structural control performance. Comparing the modal magnitude of residual modes, in these figures, one observes that spillover effects there exist, but the amplitude is small when compared with the controlled modes in figure 6a (1º and 2º modes). Figure 7 shows the physical output signal of the system.

![Figure 8. G₁ transfer function magnitude for uncontrolled and controlled system considering parametric variations, in the four vertexes (V1, V2, V3 and V4).](image)
To test the robustness characteristic to parametric variation, the system was simulated in extreme conditions. Figures 8 shows the FRF plot considering the four vertices of the polytopic system, representing the parametric variation caused by uncertainties in natural frequencies.

Obviously the controller designed based in this methodology do not assure stability because the system can become unstable face the possible spillover effects. Besides that, we have warranty of limited input in the PZTs only in the case where we have a well known initial condition. The LMIs used in the design procedure are based in invariant ellipsoid proposal, and known initial condition is an obligatory requirement to assure the saturation level in the controller, [2]. In many practical applications the initial condition is not available. So, it is need to include in the methodology a specific technique to overcome this difficulty. The next controller implemented and solved by LMI approach is used to reach the purposes of robustness face to residual dynamic.

### 3.3 Controller 2: $H_{\infty}$ output feedback control

A standard form of a general feedback system with uncertainty is given by fig. 9, where $\mathbf{w}$ is the exogenous vector, $\mathbf{z}$ is the regulated output vector, $\mathbf{y}$ is the output signal used to feedback the system, $\mathbf{K}$ is the output feedback controller, $\mathbf{u}$ is the signal of control, $\mathbf{p}$ is the output of the feedback perturbation, $\Delta$ and $\mathbf{q}$ is the input vector of the uncertainty block $\Delta$. The uncertainty block $\Delta$ represent an unstructured perturbation and it is considered an unknown variable, but with bound norm by $\|\Delta\|_\infty \leq 1$. The generalized plant $\mathbf{P}$ includes the dynamics informations of the transfer function of the system and the respectives interconnection between the input and output signals.

![Figure 9. Stand form for robust design.](image)

Mathematically the general model is given by:

$$
\begin{bmatrix}
\mathbf{q} \\
\mathbf{z} \\
\mathbf{y}
\end{bmatrix}
= 
\begin{bmatrix}
P_{wp} & P_{wq} & P_{wz} \\
P_{zp} & P_{zw} & P_{zu} \\
P_{yp} & P_{yw} & P_{yu}
\end{bmatrix}
\begin{bmatrix}
\mathbf{p} \\
\mathbf{w} \\
\mathbf{u}
\end{bmatrix}
= 
\mathbf{P}
\begin{bmatrix}
\mathbf{p} \\
\mathbf{w} \\
\mathbf{u}
\end{bmatrix}
$$

(18)

The main goal is remain the vector $\mathbf{z}$ small face to disturbances $\mathbf{w}$ and assure robustness. The classical $H_{\infty}$ theory can be used to solve this problem, [12] or [13].

**Problem:** Find a output controller $\mathbf{K}$ such that:

$$
\|\mathbf{H}_{zw}\|_\infty < \lambda \tag{19}
$$

$$
\|\mathbf{H}_{wp}\|_\infty \leq 1 \tag{20}
$$
where $\lambda$ is the cost of the design. The infinity norms above are related with the maximum gain for any transfer function, [12].

An additive uncertainty structure has been used in order to represent the unmodeled dynamic, fig. 10. Mathematically, it can be written by:

$$ G = G_c + G_r $$  \hspace{1cm} (21)

The transfer function $G_r$ is relative to residual modes. In the present example this dynamic is known, but it is very complex to use in a controller design, because the residual dynamic includes seven modes. So, a good way to overcome this difficulty it to replace it by a function with low order $G_p$. It is possible due to small gain theorem, [12]. Thus:

$$ \|G_r\|_\infty \leq \|\Delta G_r\|_\infty, \quad \text{where} \quad \|\Delta\|_\infty \leq 1 $$  \hspace{1cm} (22)

Once used a transfer function $G_p$ as the uncertainty model, which satisfies the inequality (22), the order of controller would be lower. One disadvantage of this approach is relative to a very conservative procedure, but the time of design is reduced, [13].

The problem given by equations (19) and (20) can be solved by many different ways. One of them is by using the structured singular value theory ($\mu$-synthesis). It consists in to change the standard form from robust and performance problem to a robust stability problem. However, in this procedure, the controller has high order and it can be difficult for practical implementation. There are some model reduction order
techniques that can be used to overcome it, as for instance the Moore method. Unfortunately, these concepts are not well defined in literature for problems that involve uncertainties. Despite that, some authors use classical reductions to get low order controller designed by \( \mu \)-synthesis procedure, as for example in [14].

An alternative way was proposed in [13] and [15], and it was adopted in the present paper. This approach is obtained by opening the feedback uncertainty block \( \Delta \) loop and by adding the input and output related with it in exogenous input and regulated output, respectively \( w' = [p \ w]^T \) and \( z = [q \ z]^T \), where \( z = [u' \ y']^T \).

To reduce the effect of the input \( p \) in the new regulated output an attenuation constant \( K_d \) is added to the input \( p \). In order to cancel this gain, it must be added an inverse gain in the output \( q \). Figure 11 shows the final generalized plant with the transfer function and respective input and outputs signal.

The generalized plant has the following transfer functions matrix, [15]:

\[
\begin{align*}
P_{z'w'} &= \begin{bmatrix} W_y G_d & W_y K_d \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, & P_{zu} &= \begin{bmatrix} W_y G_e \\ W_u \\ G_p G_e K_d^{-1} \end{bmatrix} \\
P_{yw} &= \begin{bmatrix} G_p & K_d \end{bmatrix}, & P_{yu} &= G_e 
\end{align*}
\]

where \( G_d \) is relative to disturbance input. The filters \( W_y \) and \( W_u \) are used to shape the regulated output. The filter \( W_y \) is often a low pass function in frequency domain and its objective is to increase the damping in the controlled modes. The weighting function \( W_u \) is chosen in order to constrain the input control signal into the interested bandwidth. Substituting eq. (23) into eq. (18), replaced \( z \) by \( z' \) and \( w \) by \( w' \), it can be obtained, after some mathematical manipulation, the following equation in closed-loop:

\[
H_{z'w'} = \begin{bmatrix} W_y & 0 & 0 \\ 0 & W_u & 0 \\ 0 & 0 & G_p K_d^{-1} \end{bmatrix} \begin{bmatrix} S & S \\ U & U \\ U & U \end{bmatrix} \begin{bmatrix} G_d & 0 \\ 0 \\ K_d \end{bmatrix}
\]

where \( S \) and \( U \) are the sensitivity and energy restriction functions, respectively. These functions are well known from classical control:

\[
S = (I - G_e K)^{-1} \tag{25}
\]

\[
U = KS \tag{26}
\]

\( W_u \) and \( G_p \) are related with \( U \), so, both functions can be used to restrict the input control signal. It occurs due the kind of unstructured uncertainty used (additive). Case one had been used another kind, as for instance multiplicative uncertainty, the results would not be equals. Another important point is to observe that \( W_u \) is more related with uncertainties in high frequencies, due, mainly, to noise measurements. Usually, the dynamic uncertainty is well known (in our example \( G_r \) is known). So, the function \( W_u \) can be omitted from the design. The new configuration of the system is given by fig. 12. In some cases it is needed to insert the attenuation gain \( K_w \) in the disturbance input to decrease the level of the exogenous signal that contribute to the response of the system, [15].

This augmented plant was done using toolbox LMI from Matlab with aid from sconnect command, [3]. The \( H_\infty \) problem, eq. (19) and (20), was solved using the command hinflmi, which applies by convex optimization technique, [3] and [5].
Figure 12. Augmented plant for robust problem used to design the controller.

3.4 Numerical application for controller 2

In order to show the applicability of the proposal approach, it was used the same example of the section 2.2. Firstly, the $W_y$ filter is chosen as a second order function with cut-off frequency between the first and the second natural modes. The unmodeled model is chosen by a low order model given by $G_p$. The values of the gains used are $K_d=0.001$ and $K_w=0.001$. The transfer functions are given by:

\[
W_y = \frac{5063}{s^2 + 270s + 202500}, \quad G_p = 0.0025 \left( \frac{\omega_1^2 s^2 + 2 \zeta_1 \omega_1 s + 1}{\omega_2^2 s^2 + 2 \zeta_2 \omega_2 s + 1} \right)^2
\]  

where $\omega_1=500 \text{ rad/s}$, $\omega_2=2700 \text{ rad/s}$, $\zeta_1=0.3$ and $\zeta_2=0.3$. Figure 13 shows the singular value plot of these functions.

Figure 13. Singular value plots for weighting functions.
The closed-loop response (FRF between disturbance input $w$ and output signal $y$) of this system in frequency domain is shown in fig. 14. There are a good amplitude attenuation in the two firsts modes. Differently of the applied approach in section 3.1, the other modes remain unaffected. The performance and robustness characteristics of the resulting controller are shown in fig. 15. The specifications given by eq. (19) and (20) are reached, because the sensitivity function is limited by inverse of the filter $W_y$ and the energy restriction function is limited by inverse of the $G_p$, as shown in fig. 15. Thus, despite of perturbations that may occurs due to residual modes, the system remains in acceptable levels of resonance peaks.

![Figure 14. FRFs of the controlled and uncontrolled response.](image)

![Figure 15. Design performance and robustness.](image)

Another important point can be understood by fig. 15b, the large peaks correspond to low frequency modes, it means that input control signal can not be excited by the residual modes. The function $U$ shows adequate robustness requirements.
4 Conclusions

The use of piezoelectric material coupled in flexible structure as actuator and sensor has shown to be a good solution in order to reduce mechanical vibration through the application of distributed controllers. Among these methodologies, the LMIs techniques (classified by some authors as postmodern control) present many advantages, mainly due the facilities to solve numericals problems without easy analytical solution. In this sense, this article presented a basic review about two populars approaches to design a robust controller for application in smart structures. The results shown the improvement of the performance of the system in the interested peaks resonance.

A possible future direction to research may include state-feedback synthesis considering norm bound as uncertainty. There are similar LMIs discussed in the literature that consider uncertainty to be norm bound. Besides this, the inclusion of the both uncertainties requirements, parametric and unmodelled dynamic robustness, in the design can be easily incorporated in the $H_\infty$ problem discussed in this paper. By the way, the proposal of solution exemplified in this paper has great advantage: the controller obtained has low order. In our example, the plant has order 18$^{th}$ (it is considered 9 naturals modes), and the controller has order 10$^{th}$ (smaller than the order of the plant in subject), four modes corresponding the controlled dynamic plus the order of the two weighting filters, one is of order $4^{th}$, $G_p$, and the another is of order $2^{nd}$, $W_y$. We should have used filters of smaller order, and in this case, the compensator would still result to be low order.

On the another hand, the use of $\mu$-synthesis procedure reaches high order for the controllers. In this case it is necessary to apply reduction order techniques for the controller, a procedure that is not well defined in the literature for uncertainty system.

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