FREQUENCY DOMAIN ANALYSIS OF DYNAMIC SYSTEMS

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Contents

Linear systems

- Continuous time invariant linear system

$$\begin{aligned}\dot{x}(t) &= Ax(t) + Bw(t)\ , \quad x(0) = 0 && (1)\\ z(t) &= Cx(t) + Dw(t) && (2)\end{aligned}$$

where $x(t) \in \mathbb{R}^n$, is the state variable, $w(t) \in \mathbb{R}^m$ is the input variable and $z(t) \in \mathbb{R}^r$ is the output variable. Matrices A, B, C and D are real matrices of compatible dimensions.

  - General solution:

$$\begin{aligned}x(t) &= \int_0^t \Phi(t,\tau)Bw(\tau)d\tau\\ &= e^{At} \star Bw(t)\end{aligned}$$

where $\Phi(t,\tau)$ is the transition matrix function

$$\Phi(t,\tau) := e^{A(t-\tau)}$$

Linear systems

- Exponential calculation

$$e^{At} = \sum_{k=0}^{\infty} \frac{(At)^k}{k!}$$

- Important property : $\Phi(\tau, \tau) = I$ for all $\tau \geq 0$ and

$$\begin{aligned} \frac{\partial \Phi}{\partial t} &= Ae^{A(t-\tau)} \\ &= A\Phi = \Phi A \end{aligned}$$

- Important input function : For any continuous function $f(t) \in \mathbb{R}^n$, the Dirac's impulse $\delta(t) \in \mathbb{R}$ is such that

$$\int_0^{\infty} f(t-\tau)\delta(\tau)d\tau = f(t) \ , \quad \forall \ t \geq 0$$

Linear systems

- Impulse response : $w(t) = d\delta(t)$ for some $d \in \mathbb{R}^m$

$$x(t) = \Phi(t,0)Bd = e^{At}Bd \ , \quad \forall \ t \geq 0$$

- Consequences of the impulse response:
  - Matrix B and vector d can always be determined to impose any *initial condition*

$$x(0) = Bd$$

  - Composition of the impulse response from the q-th input channel to the p-th output channel provides

$$z(t) = Ce^{At}B + D\delta(t) \ , \quad \forall \ t \geq 0$$

Transfer functions

- Laplace transform

$$\hat{f}(s) : \mathcal{D}(\hat{f}) \to \mathbb{C}$$

$$\hat{f}(s) := \int_{-\infty}^{+\infty} f(t) e^{-st} dt$$

  - Domain of the Laplace transform is given by

$$\mathcal{D}(\hat{f}) := \{ s \in \mathbb{C} \ : \ \hat{f}(s) \ \text{ exists} \}$$

  - The domain of $\hat{f}(s)$ strongly depends on the domain of $f(t)$.
  - $\mathcal{D}(\hat{f})$ is the "maximal" region of $\mathbb{C}$ where $\hat{f}(s)$ is analytic.

Transfer functions

- Domain calculation for $f(t)$ defined for all $t \geq 0$

$$\mathcal{D}(\hat{f}) = \{s \in \mathbb{C} \;:\; \mathrm{Re}(s) > \alpha\}$$

where α is minimized, keeping $\hat{f}(s)$ analytic inside $\mathcal{D}(\hat{f})$. In other words, all poles of $\hat{f}(s)$ must be outside $\mathcal{D}(\hat{f})$.

- Inverse Laplace transform

$$f(t) := \frac{1}{2\pi j}\int_{\Gamma} \hat{f}(s)e^{st}ds \;, \quad \forall\; t > 0$$

where Γ is any vertical line inside the domain $\mathcal{D}(\hat{f})$.

Transfer functions

- Transfer function : Applying the Laplace transform to the system (1)-(2) we obtain

$$\hat{z}(s) = G(s)\hat{w}(s)$$

where $G(s) \in \mathbb{C}^{r \times m}$ given by

$$G(s) = C(sI - A)^{-1}B + D$$

is the transfer function from the input w to the output z.

  - $G(s)$ is a rational function
  - The roots of the n-th order algebraic equation

$$\det(sI - A) = 0$$

are called poles of the transfer function $G(s)$.

Transfer functions

- The linear system (1)-(2) is asymptotically stable wherever all poles of the transfer function $G(s)$ are located in the region $\mathrm{Re}(s) < 0$ of the complex plane.
- Consequences of asymptotic stability :
  - The domain of the transfer function satisfies
    $$\mathcal{D}(G) \supset \{s \in \mathbb{C} \ : \ \mathrm{Re}(s) \geq 0\}$$
    and consequently $j\omega \in \mathcal{D}(G)$ for all $\omega \in \mathbb{R}$.
  - $G(j\omega)$ is a well defined quantity for all $\omega \in \mathbb{R}$ and is called the frequency response of the system under consideration.
  - $G(j\omega)$ is the Fourier transform of $G(t) = Ce^{At}B + D\delta(t)$ defined for all $t \geq 0$.

Transfer functions

- The sinusoidal function

$$\frac{1}{s - j\omega} = \mathcal{L}(e^{j\omega t}) \ , \quad \omega \in \mathbb{R} \ , \quad t \geq 0$$

successively applied to each input channel provides the output

$$\begin{aligned} \hat{z}(s) &= \frac{G(s)}{s - j\omega} \\ &= \frac{G(j\omega)}{s - j\omega} + E(s) \end{aligned}$$

where the poles of $E(s)$ are those of $G(s)$. Assuming the system is asymptotically stable, the steady state solution is given by

$$\hat{z}_{ss}(s) = \frac{G(j\omega)}{s - j\omega}$$

Transfer functions

- The linear system under consideration satisfies :
  - Steady state response with $d \in \mathbb{R}^m$:

$$Input \implies w(t) = de^{j\omega t}$$

$$Output \implies z(t) = G(j\omega)de^{j\omega t}$$

  - T-periodic input response with $\alpha_k \in \mathbb{C}$:

$$Input \implies w(t) = \sum_{k=-\infty}^{\infty} \alpha_k e^{j\omega_k t}$$

$$Output \implies z(t) = \sum_{k=-\infty}^{\infty} \beta_k e^{j\omega_k t}$$

where $\beta_k = G(j\omega_k)\alpha_k$ and $\omega_k = k\left(\frac{2\pi}{T}\right)$ for all $k \in \mathbb{N}$.

Transfer functions

- Important consequence : If $w(t)$ is a real signal then

$$\alpha_{-k} = \alpha_k^* \ , \quad \forall\ k \in \mathbb{N}$$

The response of a real linear system has the same property, that is

$$\beta_{-k} = \beta_k^* \ , \quad \forall\ k \in \mathbb{N}$$

$$\Downarrow$$

$$G(j\omega)^* = G(-j\omega) \quad , \quad \forall\ \omega \in \mathbb{R}$$

Norms

- Consider a vector $x \in \mathbb{C}^n$ and denote $x^\sim$ its conjugate transpose. The quantity

$$\|x\| := \sqrt{x^\sim x} = \sqrt{\sum_{i=1}^{n} |x_i|^2}$$

is the Euclidean norm of the vector $x \in \mathbb{C}^n$.

- For a trajectory $x(t) \in \mathbb{C}^n$ defined for all $t \geq 0$, it is possible to define its $\mathcal{L}_2$-norm

$$\|x\|_2 := \sqrt{\int_0^\infty \|x(t)\|^2 dt} = \sqrt{\int_0^\infty x(t)^\sim x(t) dt}$$

Parseval's theorem

- Given a trajectory $x(t) \in \mathbb{R}^n$ defined for all $t \geq 0$, is it possible to determine the norm $\|x\|_2$ from its Laplace transform $\hat{x}(s)$? For trajectories such that $0 \in \mathcal{D}(\hat{x})$, the affirmative answer to this question is given by the celebrated Parseval's theorem :

$$\|x\|_2^2 = \underbrace{\frac{1}{\pi} \int_0^\infty \|\hat{x}(j\omega)\|^2 d\omega}_{\|\hat{x}\|_2^2} \tag{3}$$

  - The proof is based on the inverse Laplace transform applied with Γ being the imaginary axis, that is

$$\begin{aligned} x(t) &= \frac{1}{2\pi j} \int_\Gamma \hat{x}(s) e^{st} ds \\ &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{x}(j\omega) e^{j\omega t} d\omega \end{aligned}$$

Parseval's theorem

- and on the calculation

$$\begin{aligned}
\|x\|_2^2 &= \int_0^\infty x(t)^\sim x(t)\,dt \\
&= \frac{1}{2\pi}\int_0^\infty x(t)^\sim \left[\int_{-\infty}^\infty \hat{x}(j\omega)e^{j\omega t}\,d\omega\right] dt \\
&= \frac{1}{2\pi}\int_{-\infty}^\infty \left[\int_0^\infty x(t)' e^{-j\omega t}\,dt\right]^* \hat{x}(j\omega)\,d\omega \\
&= \frac{1}{2\pi}\int_{-\infty}^\infty \hat{x}(j\omega)^\sim \hat{x}(j\omega)\,d\omega
\end{aligned}$$

Parseval's theorem

- Since $x(t)$ is supposed to be real

$$\hat{x}(j\omega)^* = \hat{x}(-j\omega) \quad , \ \forall\ \omega \in \mathbb{R}$$

$$\Downarrow$$

$$\begin{aligned}
\|x\|_2^2 &= \frac{1}{2\pi}\int_{-\infty}^{\infty} \hat{x}(j\omega)^{\sim}\hat{x}(j\omega)d\omega \\
&= \frac{1}{\pi}\int_{0}^{\infty} \hat{x}(j\omega)^{\sim}\hat{x}(j\omega)d\omega \\
&= \frac{1}{\pi}\int_{0}^{\infty} \|\hat{x}(j\omega)\|^2 d\omega \\
&= \|\hat{x}\|_2^2
\end{aligned}$$

Routh criterium

- Asymptotic stability: For the linear system (1)-(2) we have to decide whenever the roots of the characteristic equation

$$\det(sI - A) = s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0 = 0$$

are located in the region $\mathrm{Re}(s) < 0$ of the complex plane. Some facts are important:
  - $A \in \mathbb{R}^{n\times n}$ implies that $a_{n-1}, \cdots, a_1, a_0$ are real numbers.
  - If s is a root then s^* is also a root.
  - A necessary (but not sufficient) conditions for asymptotic stability is

$$a_{n-1} > 0 \ , \ \cdots \ , \ a_1 > 0 \ , \ a_0 > 0$$

Routh criterium

- The Routh criterion is based on the Routh array

s^n	a_n	a_{n-2}	a_{n-4}	$\cdots$
s^{n-1}	a_{n-1}	a_{n-3}	a_{n-5}	$\cdots$
s^{n-2}	b_1	b_2	b_3	$\cdots$
$\cdots$	$\cdots$			
s^1	$\cdots$			
s^0	$\cdots$			

where the next row is determined from the previous two ones as follows

$$b_1 = \frac{a_{n-1}a_{n-2} - a_n a_{n-3}}{a_{n-1}}$$

$$b_2 = \frac{a_{n-1}a_{n-4} - a_n a_{n-5}}{a_{n-1}}$$

Routh criterium

- Important result : The number of sign changes in the first column of the Routh array is equal to the number of roots in the right half part of the complex plane.

$$\Downarrow$$

  - Routh criterion : The linear system (1)-(2) is asymptotically stable if and only if the first column of the Routh array is positive.

Nyquist criterion

- The Nyquist criterion is based on the "Cauchy's Residue Theorem" applied to some function of complex variable $f(z) : \mathbb{C} \to \mathbb{C}$ defined in a domain $\mathcal{D} \subset \mathbb{C}$.
  - Analytic : The function $f(z)$ is analytic at $z_0 \in \mathcal{D}$ if the derivative $f'(z)$ exists at z_0 and at every point of some neighborhood of z_0. Hence, $f(z)$ is analytic in $\mathcal{D}$ whenever $f'(z)$ exists at every $z \in \mathcal{D}$.
  - Isolated singular point : The point $z_0 \in \mathcal{D}$ is an isolated singular point of $f(z)$ whenever $f(z)$ is analytic at every point of a neighborhood of z_0 except at the point z_0 itself. The poles are the only (finite) isolated singular points of any rational function.

Nyquist criterion

- A function $f(z)$ can be developed in Laurent's series at point $z_0 \in \mathcal{D}$ where it fails to be analytic, as for instance at an isolated singular point

$$f(z) = \sum_{i=-\infty}^{\infty} c_i (z - z_0)^i$$

  - Residues : The residue of $f(z)$ at $z_0 \in \mathcal{D}$ is given by

$$\begin{aligned} R(f, z_0) &:= c_{-1} \\ &= \frac{1}{2\pi j} \oint_C f(z) dz \end{aligned}$$

where $C \subset \mathbb{C}$ is a closed contour containing z_0 in its interior.

Nyquist criterion

- The Cauchy's Residue Theorem states that

$$\frac{1}{2\pi j}\oint_C f(z)dz = \sum_{k=1}^{r} R(f, z_k)$$

where :

- $z_1, \cdots, z_r$ are isolated singular points of $f(z)$.
- the closed contour $C \subset \mathbb{C}$ contains all points $z_1, \cdots, z_r$ in its interior.

$$\Downarrow$$

Residues can be calculated by partial decomposition of $f(z)$

Nyquist criterion

- The Cauchy's Residue Theorem is applied to prove that the following equality holds

$$\frac{1}{2\pi j}\oint_C \frac{g'(z)}{g(z)}dz = N_z - N_p \tag{4}$$

where N_z is the number of zeros of $g(z)$ inside the closed contour $C \in \mathbb{C}$ and N_p is the number of poles of $g(z)$ inside the same contour.

  - Important fact : The isolated singular points of the function

$$f(z) := \frac{g'(z)}{g(z)}$$

are the poles and the zeros of $g(z)$. Hence $f(z)$ fails to be analytic at the poles and zeros of $g(z)$ that are inside C.

Nyquist criterion

- Assume that z_0 is a zero of multiplicity m_0 of $g(z)$, located inside the closed contour C. Hence,

$$g(z) = (z - z_0)^{m_0} p(z)$$

where $p(z)$ is analytic at z_0 and $p(z_0) \neq 0$ which provides

$$f(z) = \frac{m_0}{z - z_0} + \frac{p'(z)}{p(z)}$$

However, since $p'(z)/p(z)$ is analytic at z_0 it can be developed in Taylor series yielding the conclusion that $R(f, z_0) = m_0$. Doing the same for all poles and zeros inside C we get (4).

Nyquist criterion

- The line integral in (4) can also be calculated from

$$\oint_C \frac{g'(z)}{g(z)} dz = \oint_C d\ln(g(z)) = j\arg\,(g(z))|_C$$

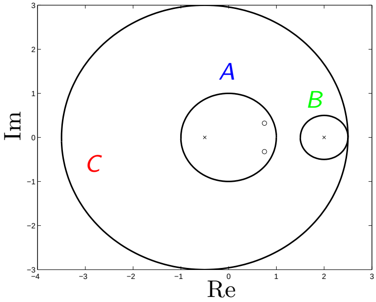
which provides the final formula

Fact (Main formula)

$$\frac{1}{2\pi}\Delta_C \; arg(g(z)) = N_z - N_p$$

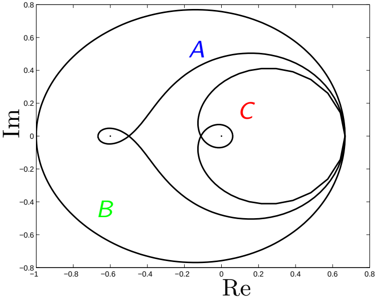
Example

- Consider the function $g(z) = \frac{1}{(z+0.5)(z-2)}$ and the closed contours A, B and C as indicated below. Notice the poles of $g(z)$ indicated by $\times$ and the zeros of $h(z) = 0.6 + g(z)$ indicated by $\circ$.



Example

- The figure below shows the closed contours obtained from A, B and C through the mapping of $g(z)$. Notice the indicated points $(0, 0)$ and $(-0.6, 0)$.



Example

- The function $g(z)$ has two poles $\{-0.5, 2\}$ and no zeros. Hence, from the contours A, B and C we have $N_z = 0, N_p = 1$, $N_z = 0, N_p = 1$ and $N_z = 0, N_p = 2$ respectively.
    - Looking at the point $(0, 0)$ we have $(1/2\pi)\Delta_A = -1$, $(1/2\pi)\Delta_B = -1$ and $(1/2\pi)\Delta_C = -2$ respectively.
- The function $h(z)$ has two poles $\{-0.5, 2\}$ and two zeros $\{0.75 \pm j0.3227\}$. Hence, from the contours A, B and C we have $N_z = 2, N_p = 1$, $N_z = 0, N_p = 1$ and $N_z = 2, N_p = 2$ respectively.
    - Looking at the point $(-0.6, 0)$ we have $(1/2\pi)\Delta_A = 1$, $(1/2\pi)\Delta_B = -1$ and $(1/2\pi)\Delta_C = 0$ respectively.

$$\Downarrow$$

Verify the main formula

Nyquist criterion

- Let us apply the previous results to the characteristic equation

$$\underbrace{s^n + a_{n-1}s^{n-1} + \cdots}_{D(s)} + \underbrace{\cdots + a_1 s + a_0}_{N(s)} = 0$$

rewritten as

$$1 + \frac{N(s)}{D(s)} = 0$$

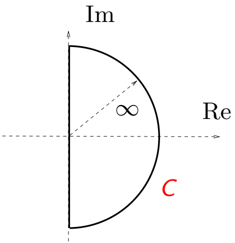
which allows us to define the rational functions

$$h(s) := 1 + g(s)\ , \quad g(s) := \frac{N(s)}{D(s)}$$

The zeros of $h(s)$ are the roots of the characteristic equation

Nyquist criterion

- Defining the closed contour C



- From the roots of $D(s) = 0$ we determine N_p, the number of poles of $h(s)$ inside C.
- From the mapping of C through $g(s)$, looking at the point $(-1, 0)$, we determine $(1/2\pi)\Delta_C \arg(h(s))$.
- Using the main formula we determine N_z, the number of zeros of $h(s)$ inside C.

Nyquist criterion

- For asymptotic stability we have to impose $N_z = 0$. Hence, denoting

$$N_{crit} := \frac{1}{2\pi}\Delta_C \arg(h(s))$$

the number of encirclements (with sign) of the mapping of the contour C through the function $g(s)$ at the critical point $(-1, 0)$ we have the celebrated :

Fact (Nyquist criterion)

The linear system (1)-(2) is asymptotically stable if and only if

$$N_{crit} + N_p = 0$$

Important notes

- Given a characteristic equation, the polynomials $N(s)$ and $D(s)$ are not unique.
- If the roots of $D(s) = 0$ are all outside C then $N_p = 0$ and the Nyquist criterion indicates that stability is possible if and only if the critical point is not encircled.
- The critical point may be any real number. Its choice depends on the particular problem under consideration.
- The contour C can be any closed contour where one wants to verify if the roots of the characteristic equation are inside to it. For instance, for discrete time systems C must be the unity circle.

Lyapunov functions

- The stability analysis of an equilibrium point $x = 0$ of a (possibly nonlinear) system with state $x(t) \in \mathbb{R}^n$ is based on the following :
  - Define a function $v(x) : R^n \to R$ given the distance of $x(t)$ at time $t \geq 0$ to the equilibrium point $x = 0$.

$$v(x) > 0 \ \forall x \neq 0 \ , \quad v(0) = 0 \ , \quad \lim_{\|x\| \to \infty} v(x) = \infty$$

  - Global asymptotic stability occurs whenever the distance decreases with respect to $t \geq 0$.

$$\dot{v}(x(t)) = \nabla_x v(x(t))' \dot{x}(t) < 0$$

Lyapunov functions

- The stability of the equilibrium point $x = 0$ of the linear system (1) with $w(t) = 0 \ , \forall t \geq 0$ and arbitrary initial condition follows from the quadratic Lyapunov function

$$v(x) = x'Px > 0 \ , \quad \forall x \neq 0 \in \mathbb{R}^n$$

and its time derivative along an arbitrary trajectory of (1)

$$\dot{v}(x) = x'(A'P + PA)x < 0 \ , \quad \forall x \neq 0 \in \mathbb{R}^n$$

Fact (Lyapunov criterion)

The linear system (1)-(2) is asymptotically stable if and only if there exists a symmetric matrix $P \in \mathbb{R}^{n \times n}$ such that

$$P > 0 \ , \quad A'P + PA < 0$$

Time varying systems

- Consider a continuous time varying linear system

$$\dot{x}(t) = A(t)x(t) \tag{5}$$

with arbitrary initial condition. Using the quadratic Lyapunov function

$$v(x(t)) = x(t)'P(t)x(t)$$

we readily obtain:

Fact (Lyapunov criterion)

The linear system (5) is asymptotically stable if and only if there exists a symmetric matrix function $P(t) \in \mathbb{R}^{n \times n}$ such that

$$P(t) > 0 \ , \quad A(t)'P(t) + P(t)A(t) + \dot{P}(t) < 0 \ , \ \forall t \geq 0$$

Time varying systems

- Important notes :
  - It is possible to impose $P(t) = P$, $\forall t \geq 0$. In this case we have to determine a symmetric matrix P such that :

$$P > 0 \ , \quad A(t)'P + PA(t) < 0 \ , \ \forall t \geq 0$$

    this simpler condition is only sufficient for asymptotic stability.
  - The Routh and Nyquist criteria do not apply.
  - The Laplace transform of (5) provides

$$s\hat{x}(s) - x_0 = \mathcal{L}(A(t)x(t))$$

    hence it is not simple (but not impossible) to determine $\hat{x}(s)$. This point will be deeply considered afterwards.

Problems

1. Consider the differential equation

$$\ddot{\theta} + 4\dot{\theta} + 4\theta = 0 \ , \quad \theta(0) = 1 \ , \ \dot{\theta}(0) = 0$$

   - Determine its solution θ and the output $\dot{\theta} + 2\theta$.
   - Determine its state space representation.
   - Determine the matrices of (1)-(2) providing the same solution from zero initial condition.

2. For a linear system with transfer function

$$G(s) = \frac{(s-2)}{(s+1)(s^2+2s+2)}$$

   Determine its impulse response.

Problems

3. Consider the transfer function

$$G(s) = \frac{s^4}{(s+1)(s+2)(s+3)(s+4)}$$

- Determine its state space representation.
- Determine the exponential function e^{At}.

4. Using Laplace transform show that for $A \in \mathbb{R}^{n\times n}$,

$$(sI - A)^{-1} = \frac{I}{s} + \frac{A}{s^2} + \frac{A^2}{s^3} + \cdots$$

Problems

5. Determine the Laplace transform and its domain for the following functions :
   - $f(t) = e^{-|t|}$ for all $-\infty < t < \infty$.
   - $f(t) = e^{-t}$ for all $0 \leq t < \infty$.
   - $f(t) = e^{t}$ for all $-\infty < t \leq 0$.
   - $f(t) = e^{t}$ for all $-\infty < t < \infty$.
   - $f(t) = e^{-t}\sin(2t)$ $0 \leq t < \infty$.
   - $f(t)$ given by the convolution of e^{-2t} and $\delta(t-2)$ defined for all $0 \leq t < \infty$.
   - $f(t) = e^{-t} + e^{t}\delta(2t)$ for all $0 \leq t < \infty$.
   - $f(t) = -(1/t)e^{-t}$ for all $0 < t < \infty$.
   - $f(t) = \mathrm{sinc}(t) = \sin(t)/t$ for all $0 < t < \infty$.
   - $f(t) = \sin^2(t)$ for all $0 < t < \infty$.

Problems

6. Given $A \in \mathbb{R}^{n\times n}$ nonsingular show that :
   - $\frac{de^{At}}{dt} = Ae^{At}$.
   - $\int_0^t e^{A\tau} d\tau = A^{-1}(e^{At} - I)$.
7. Consider a periodic input $w(t)$ with period 2 sec and a transfer function $G(s)$

$$w(t) = \begin{cases} 1, & t \in [0, 0.5) \\ 0, & t \in [0.5, 2) \end{cases}, \quad G(s) = \frac{2125}{s^3 + 15s^2 + 475s + 2125}$$

   - Determine (plot) the Fourier series of input $w(t)$.
   - Determine (plot) the Fourier series of output $z(t)$.
   - Interpret the result using the Bode plot of $G(j\omega)$.

Problems

8. Determine the domain and the inverse Laplace transform of the following functions :
    - $\hat{f}(s) = \frac{1-e^{-4s}}{s+3}$.
    - $\hat{f}(s) = \ln(s+1)$.
9. Show that if $0 \in \mathcal{D}(\hat{h})$ then

$$\frac{d}{ds}\ln(\hat{h}(s))\bigg|_{s=0} = -\frac{\int_0^\infty th(t)dt}{\int_0^\infty h(t)dt}$$

Apply and interpret this result to the functions

$$\hat{h}(s) = \frac{1}{\tau s+1}\ ,\ \ h(t) = \begin{cases} 1, & t \in [10,12] \\ 0, & t \notin [10,12] \end{cases}$$

Problems

10. For the function $f(t) = e^{-2t}$ defined for all $t \geq 0$, determine the value of the integral

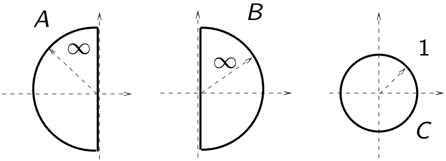
$$I = \int_0^{\infty} f(t)^2 dt$$

directly and using Parseval's theorem.

11. Using the Routh and Nyquist criteria determine the values of the parameter $\kappa \in \mathbb{R}$ such that the following algebraic equations represent asymptotic stable linear continuous time invariant systems:
    - $s^3 + 5s^2 + (\kappa - 6)s + \kappa = 0.$
    - $s(s+1)^2 + \kappa(s+4) = 0.$

Problems

12. Using the Nyquist criterion and considering the following contours A, B and C



determine, for the algebraic equations given bellow, the number of roots located inside each contour :

- $(z+0.5)(z+2)(z+4)+(z-0.5)(z-1)=0.$
- $z(z+0.5)(z+2)(z+4)+(z-0.5)(z-1)=0.$